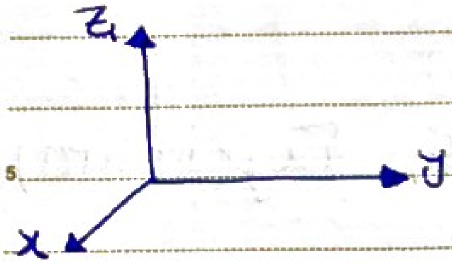
① (طراحی اجزا [illegible])

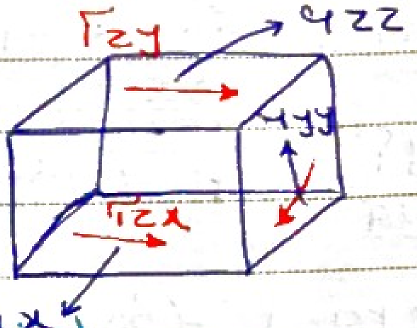
۱/۱

Subject ۷/۱۰ Year: Month: Date:

Refrence: machine Design: joseph shingley

تنش (stress): $[N/m^2]$ → secound order tensor

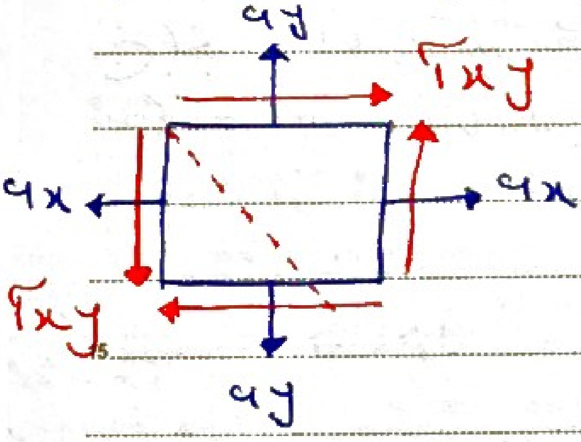


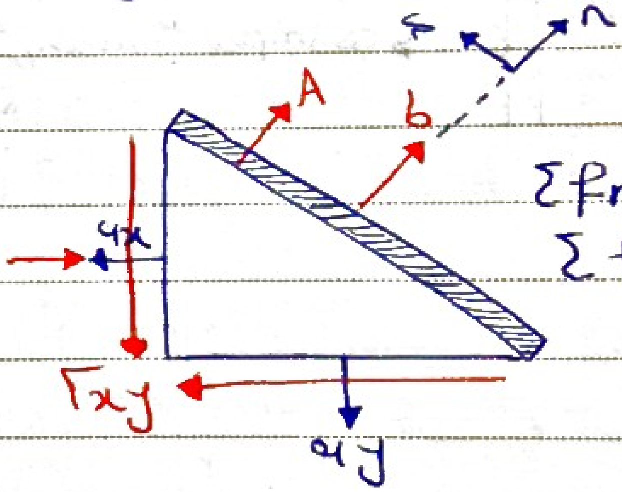


σ_{xx} → Normal stress (تنش های عمودی)

τ → shear stress (تنش های برشی)

محور اصلی: محورهای تنش که در آن τ_{max} باشد (principal axis)





$\sum f_n = 0$ ①

$\sum f_s = 0$ ②

① $\sigma A - \sigma_x A \cos^2\theta - \tau_{xy} A \cos\theta \sin\theta - \sigma_y A \sin^2\theta - \tau_{xy} A \sin\theta \cos\theta = 0$

② $\tau A + \sigma_x A \cos\theta \sin\theta - \tau_{xy} A \cos^2\theta - \sigma_y A \sin\theta \cos\theta + \tau_{xy} A \sin^2\theta = 0$

① → $\sigma = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2} \cos(2\theta) + \tau_{xy} \sin(2\theta)$

② → $\tau = -\dfrac{\sigma_x - \sigma_y}{2} \sin(2\theta) + \tau_{xy} \cos(2\theta)$

SALEH

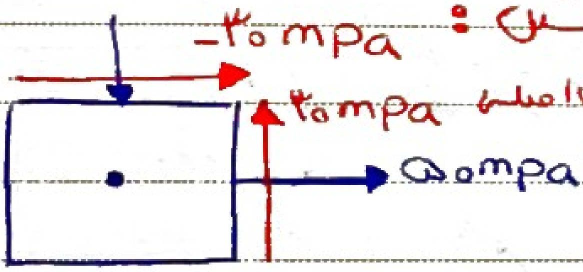
① $\Rightarrow \frac{d\sigma}{d\theta}=0 : \left(\frac{\sigma_x-\sigma_y}{2}\right)(-2\sin 2\theta)+\tau_{xy}(2\cos 2\theta)=0$

$tg(2\theta_p)=\frac{2\tau_{xy}}{\sigma_x-\sigma_y}$ $\quad \tau(\theta=\theta_p)=0$

② $\Rightarrow \frac{d\tau}{d\theta}=0 : -\left(\frac{\sigma_x-\sigma_y}{2}\right)(2\cos 2\theta)-\tau_{xy}(2\sin 2\theta)=0$

$\Rightarrow tg(2\theta_s)=\frac{-(\sigma_x-\sigma_y)}{2\tau_{xy}} \Rightarrow \sigma(\theta=\theta_s)=\frac{\sigma_x+\sigma_y}{2}$

مثال) در المان تنش نشان داده شده در شکل:



الف) جهات محورهای اصلی ب) تنش‌های اصلی

ج) جهت صفحات با تنش‌های برشی max؟

د) مقدار تنش برشی ماکسیمم؟

الف) $tg(2\theta_p)=\frac{2(30)}{50+30}=3/4 \rightarrow 2\theta_p=37^\circ , 180+37=217^\circ$

$\theta_p=18.5^\circ$

$\theta_p=108.5^\circ$ تنش ماکسیمم

ب) $2\theta=37^\circ \rightarrow \sigma_1=\frac{50+30}{2}(0.8)+\frac{50-30}{2}+30(0.6)=60mpa$

min

$2\theta=217^\circ \rightarrow \sigma_2=\frac{50-30}{2}-\frac{50+30}{2}(0.8)-30(0.6)=10-32-18=-40mpa$

$\cos(\pi+\alpha)=-\cos\alpha \quad , \quad \sin(\pi+\alpha)=-\sin\alpha$

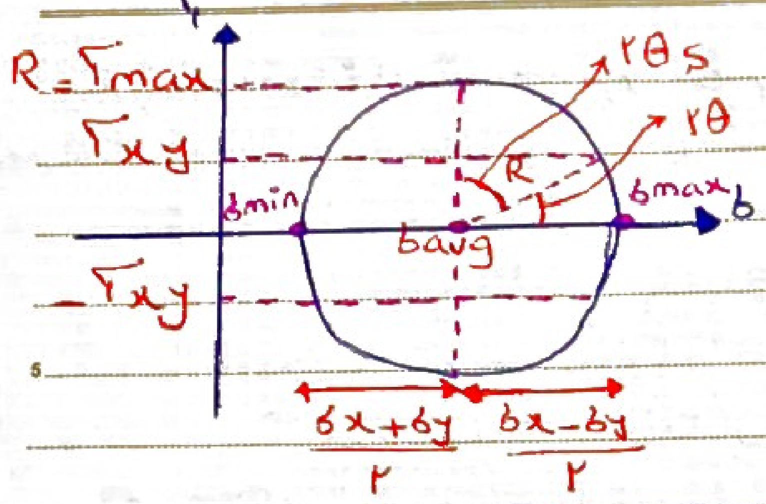
ج) $tg(2\theta_s)=\frac{-(50+30)}{2(30)}=\frac{-80}{60}=-4/3 \rightarrow 2\theta_s=-53 \rightarrow \theta_s=-26.5$

$\hookrightarrow 2\theta_s=180-53=127^\circ \quad \theta_s=63.5$

د) $2\theta_s=-53^\circ \rightarrow \tau=-\frac{50+30}{2}(-0.8)+30(0.6)=-32+18=-14mpa$

$2\theta_s=127^\circ \rightarrow \tau=\frac{50+30}{2}(0.8+30(-0.6))=14mpa$

(Ex



$$\sigma = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\tau = -\frac{\sigma_x-\sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta)$$

$$(\sigma-\sigma_{avg})^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2\cos^2(2\theta) + \tau_{xy}^2\sin^2(2\theta) + (\sigma_x-\sigma_y)\tau_{xy}\sin(2\theta)\cos(2\theta)$$

$$\tau^2 = \tau_{xy}^2\cos^2(2\theta) + \left(\frac{\sigma_x-\sigma_y}{2}\right)^2\sin^2(2\theta) - (\sigma_x-\sigma_y)\tau_{xy}\sin(2\theta)\cos(2\theta)$$

$$\Rightarrow (\sigma-\sigma_{avg})^2 + \tau^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2 \rightarrow$$ معادله دایره

$(x-\alpha)^2 + (y-\beta)^2 = R^2$ معادله دایره

مقطع = $\sigma_{max} \perp \sigma_{min}$ ؛ ۱

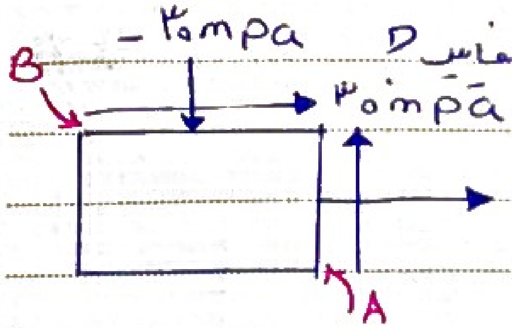
$$tg(2\theta_p) = \frac{\tau_{xy}}{\frac{\sigma_x-\sigma_y}{2}} = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

$$tg(2\theta_s) = \frac{\frac{\sigma_x-\sigma_y}{2}}{\tau_{xy}} = \frac{\sigma_x-\sigma_y}{2\tau_{xy}}$$

نتـ: $R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

SALEH

مثال) مثال قبل را با استفاده از دایره موهر حل کنید



$\sigma_x = 50\ MPa \quad \sigma_y = -30\ MPa$

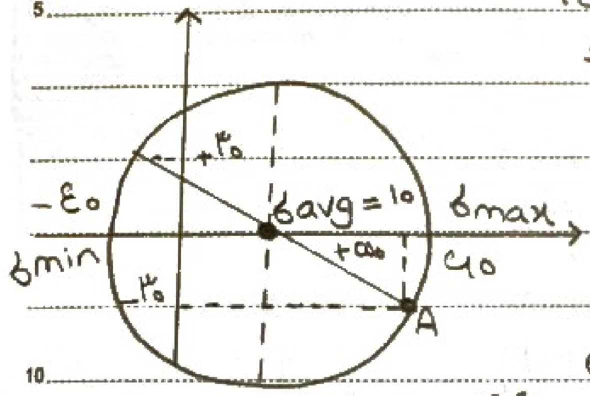
$\tau_{xy} = +40\ MPa$

$tg(2\theta_p) = 30/40 = 3/4$

$2\theta_p = 37^\circ \rightarrow \theta_p = 18.5^\circ$

$tg(2\theta) = 40/30 = 4/3$

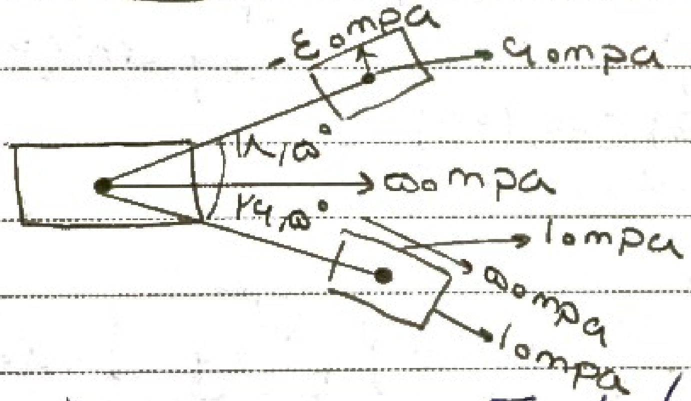
$R = \sqrt{40^2 + 30^2} = 50\ MPa$



$\sigma\ MPa \quad 2\theta_s = 53^\circ \rightarrow \theta_s = 26.5^\circ$

$\sigma_{max} = \sigma_1 = 10 + 50 = 60\ MPa$

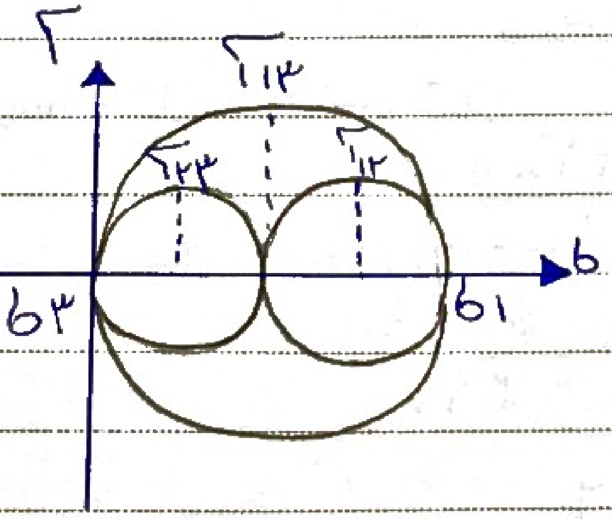
$\sigma_{min} = \sigma_2 = 10 - 50 = -40\ MPa$

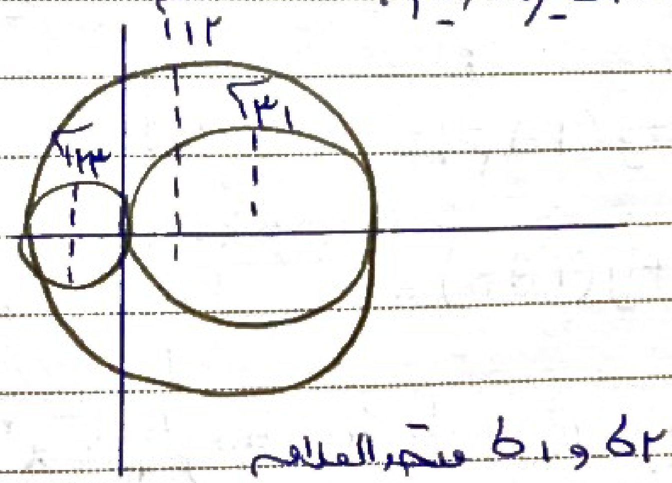


$\sigma_{max} = \sigma_{avg} + R$

$\sigma_{min} = \sigma_{avg} - R$

در حالت دو بعدی ۲ دایره داریم که یکی از آن‌ها صفر است. در حالت ۳ بعد سه دایره داریم.





σ_2 و σ_1 متحدالعلامه

$\tau_{max} = \tau_{13} = \sigma_1/2$

$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2} \quad \tau_{23} = \frac{\sigma_2 - \sigma_3}{2}$

$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2}$

SALEH

Subject Year: Month: Date:

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \quad I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$$

$$I_2 = \begin{vmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{xy} & \sigma_{yy} \end{vmatrix} + \begin{vmatrix} \sigma_{yy} & \tau_{yz} \\ \tau_{zy} & \sigma_{zz} \end{vmatrix} + \begin{vmatrix} \sigma_{xx} & \tau_{xz} \\ \tau_{zx} & \sigma_{zz} \end{vmatrix}$$

$$I_3 = \begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{vmatrix}$$

$I_1, I_2, I_3 \rightarrow$ stress invariouts

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0 \rightarrow \sigma_1, \sigma_2, \sigma_3$$

مدول یانگ young modul $E = \sigma / \varepsilon$

poisson ratio $= \dfrac{-\text{latem l strain (کرنش عرضی)}}{\text{axist strain (کرنش جانبی)}}$ $\quad 0 < \nu < 1/2$

$\sigma = \dfrac{\tau \text{ (تنش برشی)}}{\gamma \text{ (کرنش زاویه‌ای)}}$ $\qquad \sigma = \dfrac{E}{2(1+\nu)}$

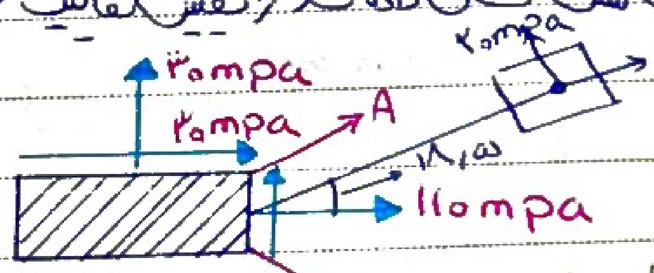
type of stress	principal strain	principal stress
uni_axial	$\varepsilon_1 = \sigma_1/E$ $\varepsilon_2 = \frac{-\nu\sigma_1}{E}$ $\varepsilon_3 = \frac{-\nu\sigma_1}{E}$	$\sigma_1 = E\varepsilon_1$ $\sigma_2 = 0$ $\sigma_3 = 0$
bi_axial	$\varepsilon_1 = \sigma_1/E - \frac{\nu\sigma_2}{E}$ $\varepsilon_2 = \sigma_2/E - \frac{\nu\sigma_1}{E}$ $\varepsilon_3 = -\frac{\nu\sigma_1}{E} - \frac{\nu\sigma_2}{E}$	$\sigma_1 = \frac{E\varepsilon_1 + \nu E\varepsilon_2}{1-\nu^2}$ $\sigma_2 = \frac{E\varepsilon_2 + \nu E\varepsilon_1}{1-\nu^2}$ $\sigma_3 = 0$
Tri_axial	$\varepsilon_1 = \sigma_1/E - \frac{\nu\sigma_2}{E} - \frac{\nu\sigma_3}{E}$ $\varepsilon_2 = \sigma_2/E - \frac{\nu\sigma_1}{E} - \frac{\nu\sigma_3}{E}$ $\varepsilon_3 = \sigma_3/E - \nu\,\sigma_1/E - \frac{\nu\sigma_2}{E}$	$\sigma_1 = \frac{E\sigma_1(1-\nu) + \nu E(\varepsilon_2+\varepsilon_3)}{1-\nu-2\nu^2}$ $\sigma_2 = \frac{\sigma E_2(1-\nu) + \nu E(\varepsilon_1+\varepsilon_3)}{1-\nu-2\nu^2}$ $\sigma_3 = \frac{\sigma E_3(1-\nu) + \nu E(\varepsilon_1+\varepsilon_2)}{1-\nu-2\nu^2}$

مثال) با استفاده از دایره موهر در المان تنش نشان داده شده را تعیین کنید: D

الف) تنش‌های اصلی

ب) جهات اصلی

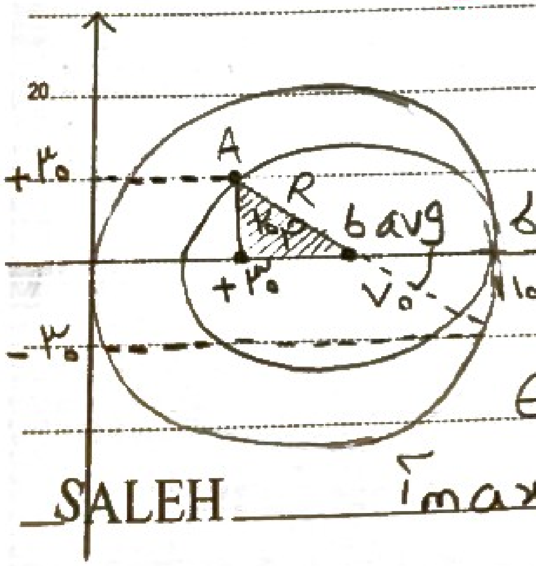
ج) تنش برشی max



$R = \sqrt{30^2 + 40^2} = 50$

$\sigma max = \sigma avg + R = 70 + 50 = 120$

$\sigma min = \sigma avg - R = 70 - 50 = 20$



$tg(2\theta p) = \frac{30}{40} = 3/4 \Rightarrow 2\theta p = 37°$

$\theta = p = 18.5$

$\tau max = \frac{120 - 0}{2} = 60 mpa$ دایره بزرگتر

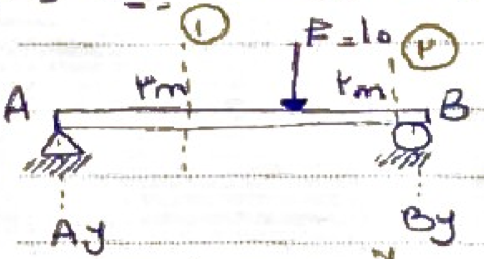
(۷)

Subject ۹۸/۷/۱۷
Date

«طراحی اجزاء در بیومکانیک»

دیاگرام نیروی برشی و گشتاور خمشی:

مثال) در تیر بارگذاری شده در شکل تغییرات نیروی برشی و گشتاور خمشی را نقش نمایید؟

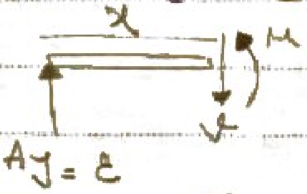


$\sum F_x = 0 \longrightarrow A_x = 0$

$\sum F_y = 0 \longrightarrow A_y + B_y = 10 \longrightarrow A_y = 4\,kN$

$\sum M_B = 0 \longrightarrow -A_y + 10(2) \rightarrow A_y = +4\,kN$

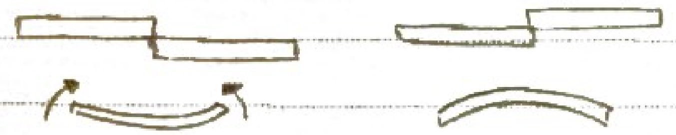
(1) $0 < x < 3$



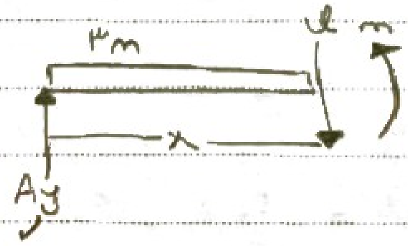
$\sum F_y = 0$

$4 - V = 0 \longrightarrow V = 4\,kN$

$\sum M_P = 0 \longrightarrow M - 4x = 0 \longrightarrow M = 4x$



(2) $3 < x < 5$



$\sum F_y = 0 \longrightarrow 4 - 10 - V = 0 \longrightarrow V = -6$

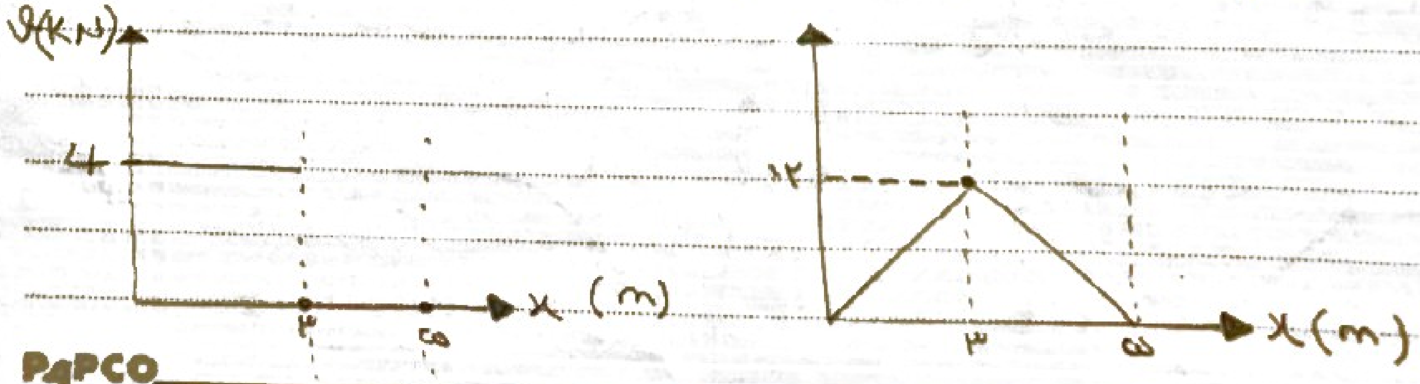
$\sum M = 0 \longrightarrow M - 4x + 10(x-3) = 0$

$M = -6x + 30$

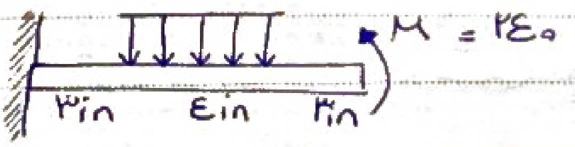
راه حل جایگزین $3 < x < 5$

$\sum F_y = 0 \longrightarrow V + 6 = 0 \longrightarrow V = -6$

$\sum M_P = 0 \longrightarrow M - 6(5-x) = 0 \longrightarrow M = -6x + 30$



مثال) در تیر بارگذاری شده در شکل زیر تغییرات نیروی برشی و گشتاور خمشی را رسم کنید؟

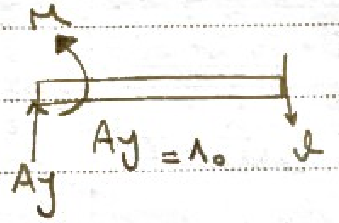


$\Sigma F_x = 0 \longrightarrow A_x = 0$

$\Sigma F_y = 0 \longrightarrow A_y - 80 = 0 \longrightarrow A_y = 80$

$\Sigma M_A = 0 \longrightarrow M_A \longrightarrow M_A - 80 + 240 = 0 \longrightarrow M_A = +160$

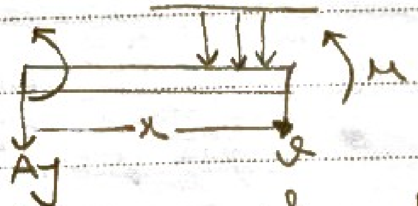
$0 < x < 3$



$\Sigma F_y = 0 \longrightarrow 80 - V = 0 \longrightarrow V = 80$

$\Sigma M_p = 0 \longrightarrow M + 160 - 80x = 0 \longrightarrow M = 80x - 160$

$3 < x < 7$

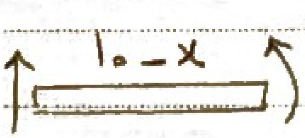


$\Sigma F_y = 0$

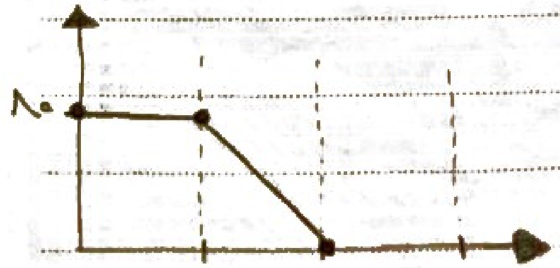
$\longrightarrow 80 - 20(x-3) - V = 0 \longrightarrow V = -20x + 140$

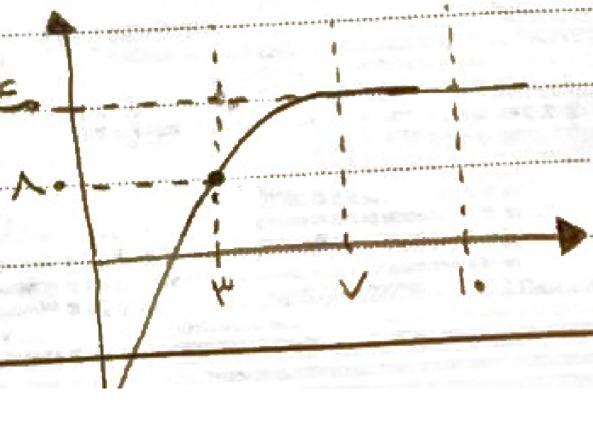
$\Sigma M_p = 0 \longrightarrow M + 20(x-3)\frac{x-3}{2} - 80x + 160 = 0$

$M = -10(x-3)^2 + 80x - 160 \longrightarrow M = -10x^2 + 140x - 250$



$\Sigma F_y = 0 \longrightarrow V = 0 \quad \Sigma M_p = 0 \longrightarrow M - 240 = 0 \longrightarrow M =$





$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \quad I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$$

$$I_2 = \begin{vmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{xy} & \sigma_{yy} \end{vmatrix} + \begin{vmatrix} \sigma_{yy} & \tau_{yz} \\ \tau_{zy} & \sigma_{zz} \end{vmatrix} + \begin{vmatrix} \sigma_{xx} & \tau_{xz} \\ \tau_{zx} & \sigma_{zz} \end{vmatrix}$$

$$I_3 = \begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{vmatrix}$$

$I_1, I_2, I_3 \rightarrow$ stress invariouts

$$\sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = 0 \rightarrow \sigma_1, \sigma_2, \sigma_3$$

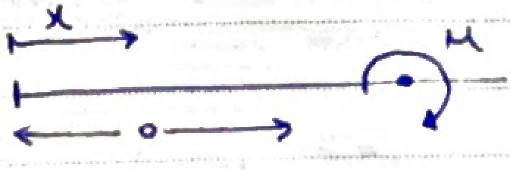
مدول یانگ young modal $E = \sigma / t$

Poisson ratio $=$ −latem l strain کرنش عرضی / axist strain کرنش جانبی

$\sigma = \dfrac{\tau}{\gamma}$ تنش برشی / کرنش زاویه‌ای $\qquad \sigma = \dfrac{E}{2(1+\nu)} \qquad 0 < \nu < 1/2$

توابع تکین (Singular Function)

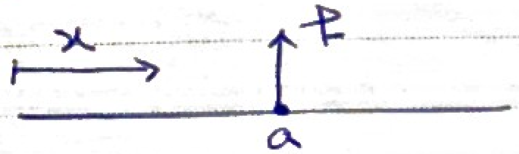
۱ گشتاور متمرکز (concenttauted moment)



$$g(x) = \langle x-a\rangle^{-2} = \begin{cases} \pm\infty \; ; & x=a \\ 0 \; ; & x\neq a \end{cases}$$

$$\int \langle x-a\rangle^{-2} dx = \langle x-a\rangle^{-1}$$

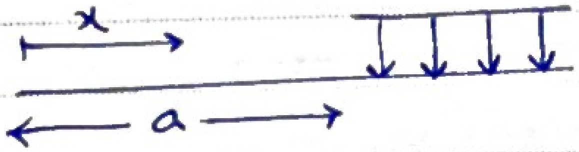
۲ Concentuated Force



$$g(x) = \langle x-a\rangle^{-1} = \begin{cases} +\infty \; ; & x=a \\ 0 \; ; & x\neq a \end{cases}$$

$$\int \langle x-a\rangle^{-1} dx = \langle x-a\rangle^{0}$$

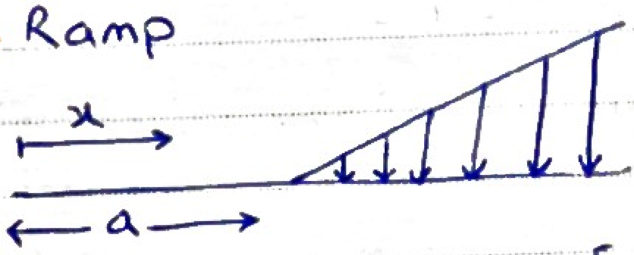
۳ step



$$g(x) = \langle x-a\rangle^{0} = \begin{cases} 1 \; ; & x\geq a \\ 0 \; ; & x< a \end{cases}$$

$$\int \langle x-a\rangle^{0} dx = \langle x-a\rangle^{1}$$

۴ Ramp

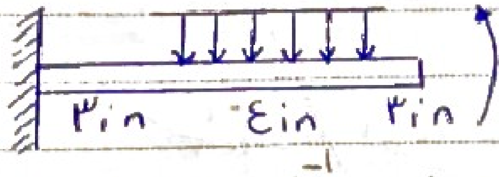


$$g(x) = \langle x-a\rangle^{-1} = \begin{cases} x-a \; ; & x\geq a \\ 0 \; ; & x< a \end{cases}$$

$$\int \langle x-a\rangle^{1} dx = \tfrac{1}{2} \langle x-a\rangle^{2}$$

Subject Year: Month: Date:

مثال) مثال دوم سری پیش را با استفاده از روش تکین حل کنید.



$M = 240$

$q(x) = A_y\langle x\rangle^{-1} - M_A\langle x\rangle^{-2} - 20\langle x-3\rangle^{0} + 20\langle x-7\rangle^{0} - 240\langle x-10\rangle^{-2}$

$V(x) = A_y\langle x\rangle^{0} - M_A\langle x\rangle^{-1} - 20\langle x-3\rangle^{1} + 20\langle x-7\rangle^{1} - 240\langle x-10\rangle^{0}$

$M(x) = A_y\langle x\rangle^{1} - M_A\langle x\rangle^{0} - 10\langle x-3\rangle^{2} + 10\langle x-7\rangle^{2} - 240\langle x-10\rangle^{0}$

$M(10+\varepsilon) = 0 = 80(10) - M_A(1) - 10(10-3)^2 - 10(10-7)^2 - 240 \rightarrow 0 = 800 - M_A - 490 + 90 - 240 \rightarrow M_A = 160\,lb.in$

$0<x<3 \rightarrow V(x) = 80\,lbs \rightarrow M(x) = 80x - 160$

$3<x<7 \rightarrow V(x) = 80 - 20(x-3) = -20x + 140 \rightarrow$

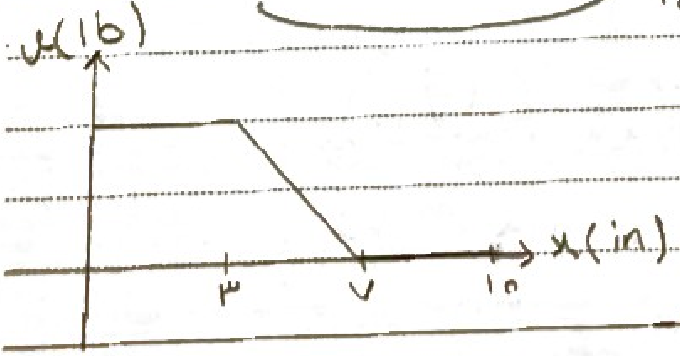
$M(x) = 80x - 160 - 10(x-3)^2$

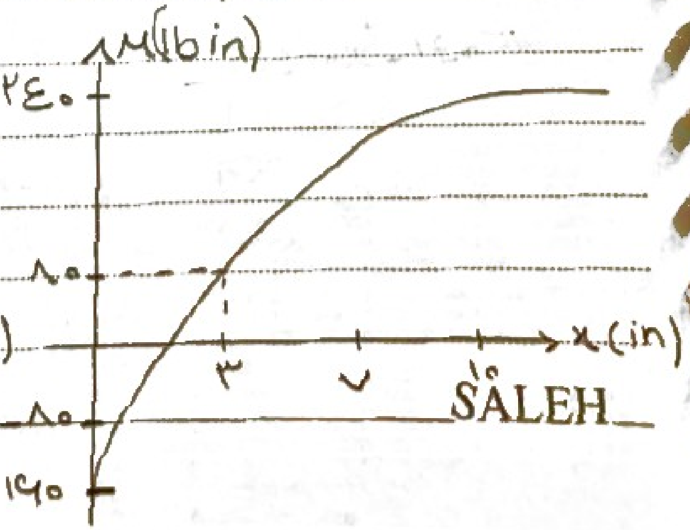
$M(x) = 80x - 160 - 10x^2 + 60x - 90 = 10x^2 + 140x - 250$

$7<x<10 \rightarrow V(x) = 80 - 0 - 20(x-3) + 20(x-7) = 0$

$M(x) = 80x - 160 - 10(x-3)^2 + 10(x-7)^2 = 80x - 160 - 10x^2 + 60x - 90 + 10x^2 - 140x + 490$

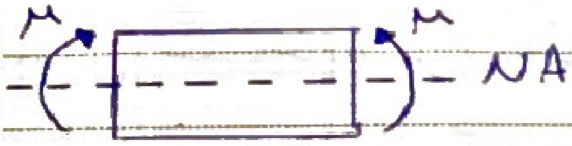
$M(x) = +240\ lb.in$



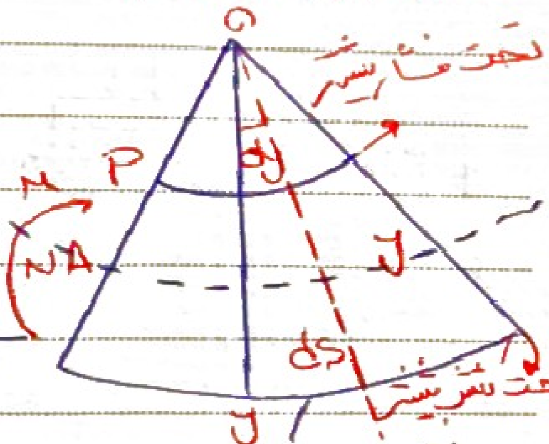


SALEH

Subject   Year: Month: Date:



خمش (Bending)



$$ds = \rho\, d\phi$$

$$tg(d\phi) \simeq d\phi = \frac{dx}{y}$$

$$dx = -y\, d\phi$$

$$\varepsilon = \frac{dx}{ds} = \frac{-y\, d\phi}{\rho\, d\phi} \longrightarrow \varepsilon = \frac{-y}{\rho}$$

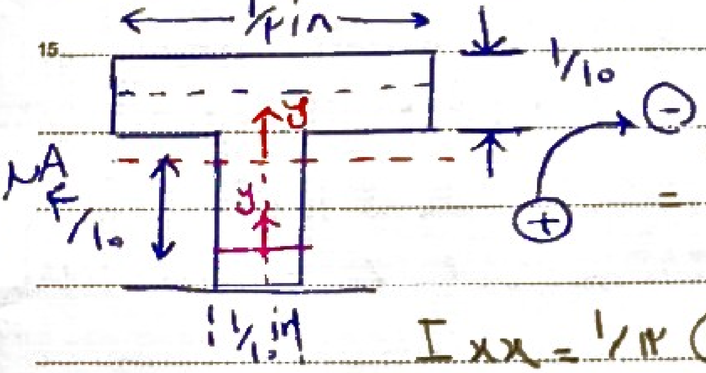
$$\sigma = \varepsilon E \longrightarrow \sigma = \frac{-Ey}{\rho}$$

تار خنثی روی مرکز هندسی مقطع قرار دارد.

$$\int \sigma y\, dA = M \longrightarrow \int -Ey/\rho\; y\, dA = M \rightarrow -E/\rho \int y^2\, dA = M$$

$$-E/\rho\; \alpha I_{xx} = M \longrightarrow \frac{\sigma}{y} I_{xx} = M \longrightarrow \sigma = \frac{My}{I_{xx}}$$

مثال) اگر در مثال قبل مقطع مقطع تیر معلق شکل زیر باشد تعیین نمایید حداکثر تنش عمودی متناظر با کشش و فشار.



$$\bar{y} = \frac{\Sigma y \cdot A}{\Sigma A} = \frac{1/20\,(1/4) + 1/20\,(1/2 + 1/20)}{1/2 \times 1/10 + 1/2 \times 1/10}$$

$$= \frac{1/20\,(1/4 + 11/20)}{2/20} = 1/2 \times 16/20 = 4/10$$

$$I_{xx} = 1/12\,(1/2)^3(1/100) + 1/20\,(3/20)^2 + 1/12\,(1/10)^3(1/2)$$

$$+ 1/20\,(3/20)^2 \qquad I_{xx} = 1/12\, bH^3$$

$$I = I_{xx} + Ad^2 \longrightarrow 240\ \text{lbin}$$

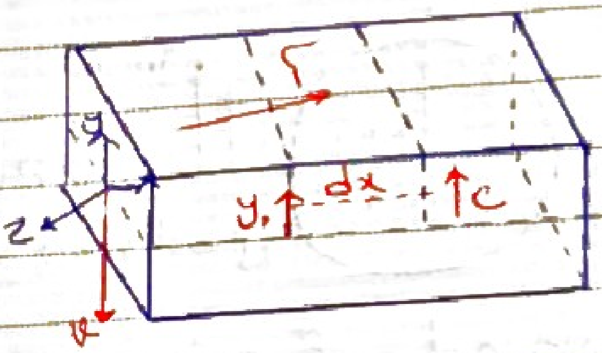
$$\sigma_C = \frac{My_C}{I_{xx}} = \frac{-M}{I_{xx}}\,(2/10)$$

$$\sigma_t = \frac{My}{I_{xx}} = \frac{+M}{I_{xx}}\,(4/10)$$

Subject   Year: Month: Date:

Shear Bending (برش همراه خمش)

$$\sigma = \frac{My}{I}$$



$$\Sigma F_z = 0 \rightarrow F_2 - F_1 = \tau b\,dx$$

$$F_2 = \int_{y_1}^{c} \frac{(M+dM)y}{I}\,dA$$

$$F_2 = \frac{M+dM}{I}\int_{y_1}^{c} y\,dA$$

$$F_1 = \int_{y_1}^{c} \frac{My}{I}\,dA \qquad F_2 = \int_{y_1}^{c} \frac{(M+dM)}{I}\,dA$$

$$F_1 = \frac{M}{I}\int_{y_1}^{c} y\,dA \qquad F_2 = \frac{M+dM}{I}\int_{y_1}^{c} y\,dA$$

$$F_2 - F_1 = \frac{dM}{I}\int_{y_1}^{c} y\,dA = \tau b\,dx$$

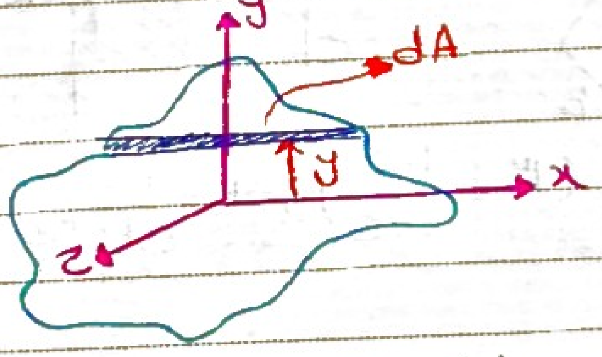
$$\tau = \left(\frac{dM}{dx}\right)\frac{\int_{y_1}^{c} y\,dA}{Ib}$$

← نیروی برشی

گشتاور اول مساحت :

$$\int y\,dA = Q$$

$$\tau = \frac{VQ}{Ib}$$



مثال) تغییرات تنش برشی ناشی از خمش در تیری با سطح مقطع مستطیلی شکل را محاسبه نمایید.

$$Q = \int_{y_1}^{h/2} y\,b\,dy \rightarrow Q = \frac{1}{2}\times b\left(\frac{h^2}{4} - y_1^2\right)$$

$$\tau = \frac{V \times \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)}{\frac{1}{12}\times bh^3 \times b} \longrightarrow \tau = \frac{6V\left(\frac{h^2}{4} - y_1^2\right)}{bh^3}$$

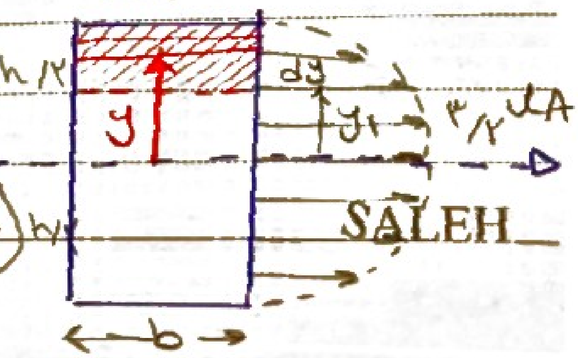
$$\rightarrow \tau = \frac{6V}{A}\left(\frac{1}{4} - \left(\frac{y_1}{h}\right)^2\right)$$

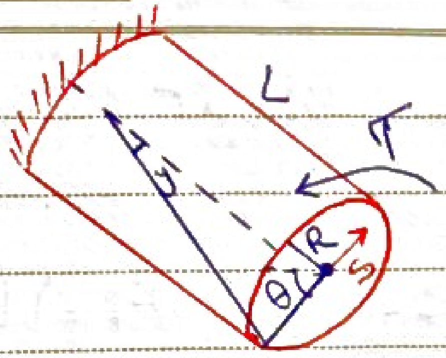
$$\tau = \frac{3V}{2A}\left(1 - 4\left(\frac{y_1}{h}\right)^2\right)$$

$$1 - 4\left(\frac{1}{4}\right) = 0 \rightarrow \left(\frac{y_1}{h}\right)^2 = \frac{1}{4} \rightarrow \frac{y_1}{h} = \frac{1}{2}$$

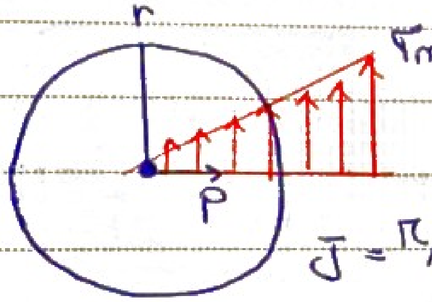
$$\tau_{max} = \frac{4}{3}\frac{V}{A}$$

$$\tau_{max}(y_1 = 0) = \frac{3V}{2A}$$





پیچش (Tonsion):

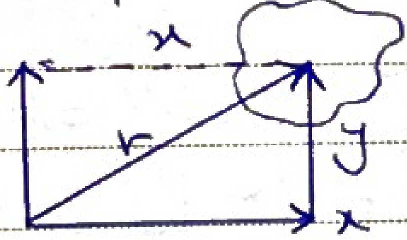


$0 < \rho < r$

$$\tau = \frac{T\rho}{J}$$

$$J = \frac{\pi}{2} r^4 = \frac{\pi r^4}{32}$$

J → polar moment second of area



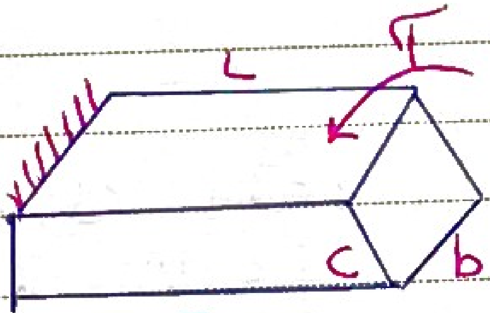
$r^2 = x^2 + y^2$, $I_{xx} = \int y^2 \, dA$

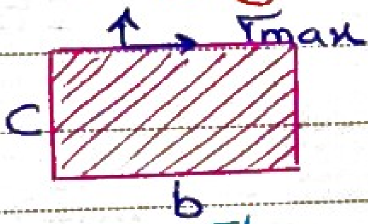
$I_{yy} = \int x^2 \, dA$

$J = \int r^2 \, dA = I_{xx} + I_{yy}$

$\tau_{max} = Tr/J$, $\theta = \frac{TL}{GJ}$ → G: shear modulus

پیچش در مقاطع غیر دایره‌ای:



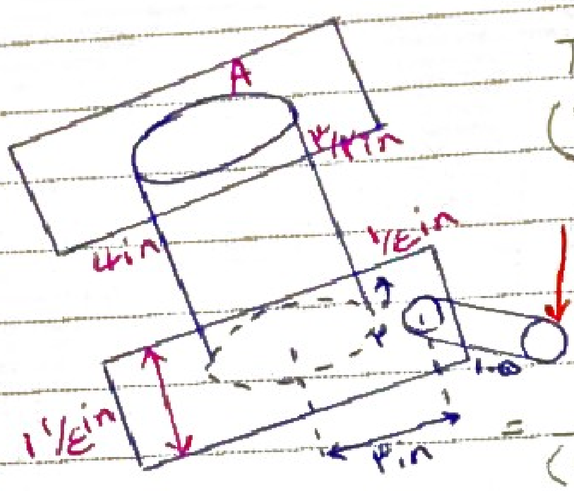


$$\tau_{max} = \frac{T}{\alpha b c^2} = \frac{T}{bc^2}\left(3 + \frac{1.8}{b/c}\right), \quad \theta = \frac{TL}{\beta b c^3 G}, \quad \beta \Rightarrow b/c$$

(الف)

شکل) مقدار تنش برشی ماکسیمم را در عضو BC از سازه نشان داده شده در شکل تعیین نمایید؟

ب) تنش عمودی max و برشی ماکسیمم را در نقطه A تعیین نمایید؟



$$T_{BC} = 300\,lb \times 1.5\,in = 450\,lb.in$$

$$(\tau_{max})_{bc} = \frac{T}{\alpha b c^2} = \frac{T}{bc^2}\left(3 + \frac{1.8}{b/c}\right)$$

$$b = 1\tfrac{1}{4} = 1.25\,in$$

$$c = 1/4 = 0.25\,in$$

$$= \frac{450\,lb.in}{(1.25)(0.25)^2}\left(3 + \frac{1.8}{1.25/0.25}\right) = 5.784\,KIP$$

$$M_{BC} = 300\,lb \times 3\,in = 900\,lb.in \;(i)$$

$$(\sigma_{max})_{bc} = \frac{MC}{I} = \frac{900\,lbin \times 1.25/2}{1/12\,(0.25)(1.25)^3} = 17.28\,KIP\;(Kpsi)$$

c, 0.25, 1.25

$$*psi = \frac{lb}{in^2}$$

$$M_{bA} = 450 + (300)(4) = 1650\,lbin\;(+j)$$

$$T_{AB} = 900\,lb.in$$

A

$$\sigma_A = \frac{MC}{I} = \frac{1650\,lbin \times 3/4\,in}{\pi/4\,(3/4)^4} = 4.98$$

$$\tau_A = Tr/J = \frac{900\,lbin \times 3/4\,in}{\pi/2\,(3/4)^4} = 2.72\,KIP$$

$$(\sigma_{max})_A = \sigma_{avg} \mp R = \frac{\sigma_x + \sigma_y}{2} \mp \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{4.98 + 0}{2} \mp \sqrt{\left(\frac{4.98 - 0}{2}\right)^2 + (2.72)^2}$$

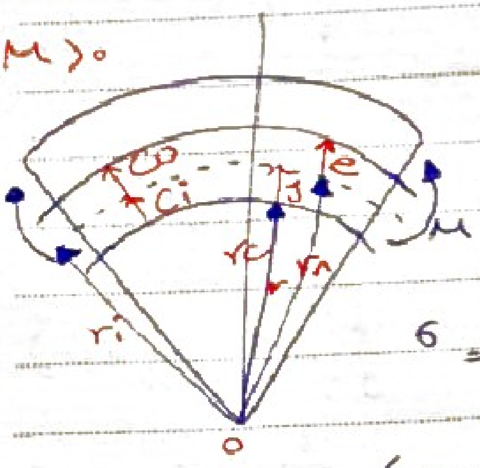
SALEH

$$\tau_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{4.98}{2}\right)^2 + (2.72)^2}$$

Subject / Date: ۹۸,۸,۲۴

Bending in curved members

$M > 0$



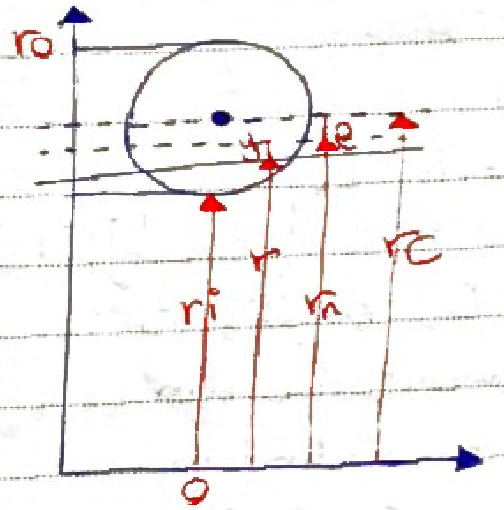
$$r + y = r_n$$

$$y = r_n - r$$

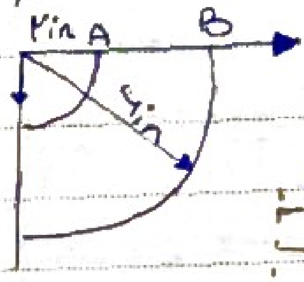
$$r_n = \frac{A}{\int_{r_i}^{r_o} \frac{dA}{r}} \qquad e = r_c - r_n > 0$$

$$\sigma = \frac{My}{Ae(r_n - y)} = \frac{M(r_n - r)}{Aer}$$

$$\sigma_{max} = \frac{MC_i}{Aer_i} \qquad \sigma_{min} = \frac{MC_o}{Ar_o}$$



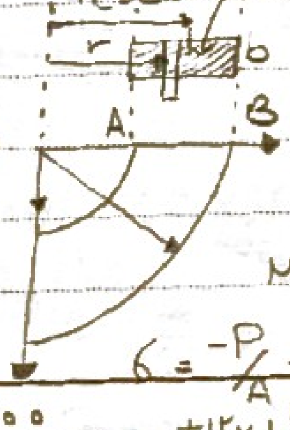
مثال) تغییرات تنش در مقطع A و B از عضو خمیده‌ی نشان داده شده در شکل را تحت بار ۳۰۰۰ به دست آورید.



$\frac{3}{4}$ in

$dA = b\,dr$

$$r_n = \frac{3/4 \times 4}{\int_2^6 \frac{3/4\,dr}{r}} = \frac{4}{\ln(6/2)} = 3.64$$



$$e = r_c - r_n = 4 - 3.64 = 0.36$$

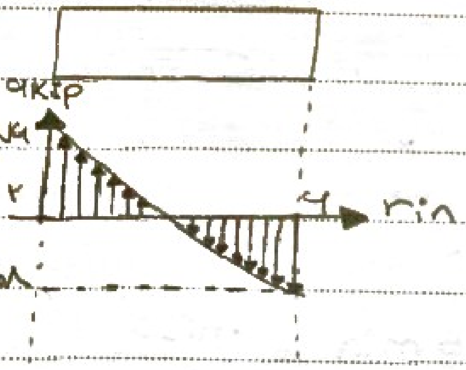
$$M = 3000\,lb \times 4\,in = 12 \times 10^3\ lb \times in$$

$$\sigma = -\frac{P}{A} + \frac{M(r_n - r)}{Aer}$$

$$O = \frac{-3000}{2\,in^2} + \frac{+12 \times 10^3 (3.64 - r)}{2\,in \times 0.36 \times r} = -1\,KIP + 11.74\,\frac{3.64 - r}{r}$$

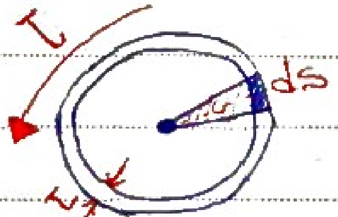
$\sigma_T = -1\,KIP + ۱۱,۷۶\ \frac{۳,۴۴ - ۲}{۲}\ KIP = ۸,۷۶\ KIP$

$\sigma_C = -1\,KIP + ۱۱,۷۶\ \frac{۳,۴۴ - ۶}{۶} = -۵,۵۸\ KIP$



□ تنش برشی در مقاطع جدار نازک :

t = ضخامت   τ = تنش برشی   t , τ = ثابت



$\int r \tau t\, ds = T$

$\longrightarrow (t\tau)\underbrace{\int r\, ds}_{2A} = T \longrightarrow \boxed{\tau = \frac{T}{2tA}}$

t ضخامت   T گشتاور پیچشی

Am مساحت احاطه شده به وسیله خط میانی

$\theta = \frac{T L_m L}{4 A_m^2 G t}$ Lm طول میانه

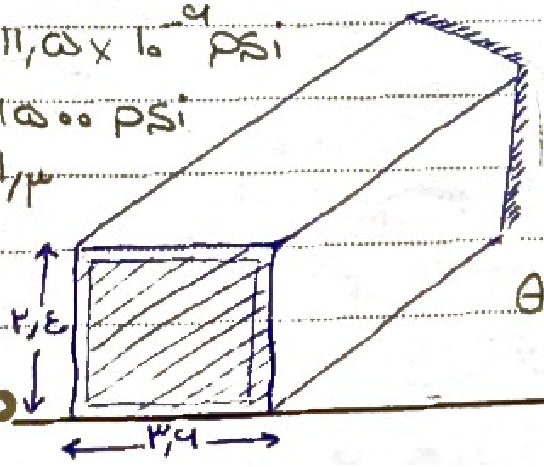
L طول مقطع

اگر تنش برشی مجاز در سازه نشان داده شده ۱۱۵۰۰ psi باشد با توجه به هندسه مقطع نشان داده شده max گشتاور پیچشی مجاز و زاویه پیچش متناظر آن ؟

$G = ۱۱,۵ \times 10^6\ psi$

$\tau = ۱۱۵۰۰\ psi$

$t = ۱/۸$



$\tau = \frac{T}{2tA_m} \longrightarrow T = 2tA_m \times \tau$

$A_m = (۳,۶ - ۰,۱۲۵)(۲,٤ - ۰,۱۲۵)$

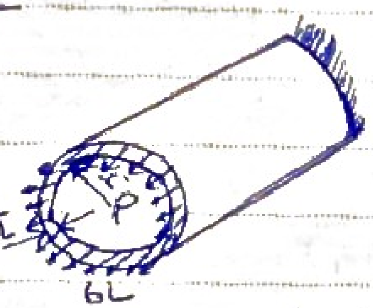
$A_m = ۷,۹\ in^2$

$T = ۲(۰,۱۲۵)(۷,۹) \times ۱۱۵۰۰ = ۲۲۷۱۲\ lb\,in$

$\theta = \frac{T L_m L}{4 A_m^2 G t} = \frac{۲۲۷۱۲ \times ۲(۳,۶ - ۰,۱۲۵)(۲,٤ - ۰,۱۲۵)}{4\,(۷,۹)^2 \times ۱۱,۵ \times 10^6 \times ۰,۱۲۵}$

$$\frac{t}{r_i} < \frac{۱}{۱۲}$$

□ تنش در استوانه‌های جدار نازک:



تنش طولی

$$p_i \times \pi r_i^{۲} = \sigma_L \times (۲\pi r) \times t$$

$$\sigma_L = \frac{p_i r_i}{۲t}$$

$$p_i(۲r_i \times L) = \sigma_t \times (۲tL)$$

$$(\sigma_t)_{avg} = \frac{P d_i}{۲t} \qquad (\sigma)_{max} = \frac{P(d_i + t)}{۲t}$$

$$\sigma_L = \frac{p_i d_i}{4t}$$

مثال) تنش‌های طولی و محیطی استوانه‌ای را به قطر خارجی ۴in و ضخامت ۱/۸ in تعیین کنید اگر فشار داخل استوانه ۱۰۰psi باشد؟

$d_o = ۴in$ و $t = ۱/۸$

$$d_i = d_o - ۲t = ۴ - ۲(۱/۸) = ۳٫۷۵$$

$$\longrightarrow r_i = \frac{d_i}{۲} = \frac{۳٫۷۵}{۲} = ۱٫۸۷۵$$

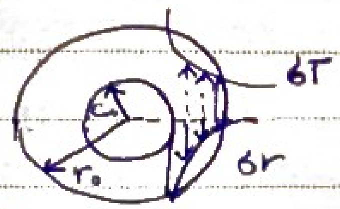
$$\frac{t}{r_i} = \frac{۰٫۱۲۵}{۱٫۸۷۵} = ۰٫۰۶۷ \longrightarrow ۰٫۰۸۳ \longrightarrow ۱/۱۲$$

$$(\sigma_t)_{max} = \frac{p_i(d_i + t)}{۲t} = \frac{۱۰۰(۳٫۷۵ + ۰٫۱۲۵)}{۲ \times ۰٫۱۲۵}$$

$$(\sigma_t)_{max} = ۱۵۵۰ psi$$

$$(\sigma_L) = \frac{p_i d_i}{4t} = \frac{۱۰۰ psi \times ۳٫۷۵ in}{۴ \times ۰٫۱۲۵ in} = ۷۵۰ psi$$

تنش در لوله‌های جدار ضخیم:



$$\sigma r = \frac{P_i\, r_i^2}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right)$$

$$\frac{t}{d_i} > \frac{1}{12}$$

$$\sigma T = \frac{P_i\, r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

$$\sigma L = \frac{P_i\, r_i^2}{r_o^2 - r_i^2}$$

مثال) اگر فشار سیستول در آئورت ۱۲۰ میلی متر جیوه و قطر بیرونی آن ۲.۷cm و ضخامت آن ۴mm باشد تعیین نمایید تغییرات تنش شعاعی و مماسی در ضخامت آئورت با فرض این که همه به صورت جدار ضخیم زده شود.

$d_o = 2.7\,cm$

$t = 4\,mm$ $\qquad d_i = d_o - 2t \rightarrow 27 - 2(4) = 19\,mm$

$$\frac{t}{d_i} = \frac{4}{19\,mm} = 0.21 > 0.082$$

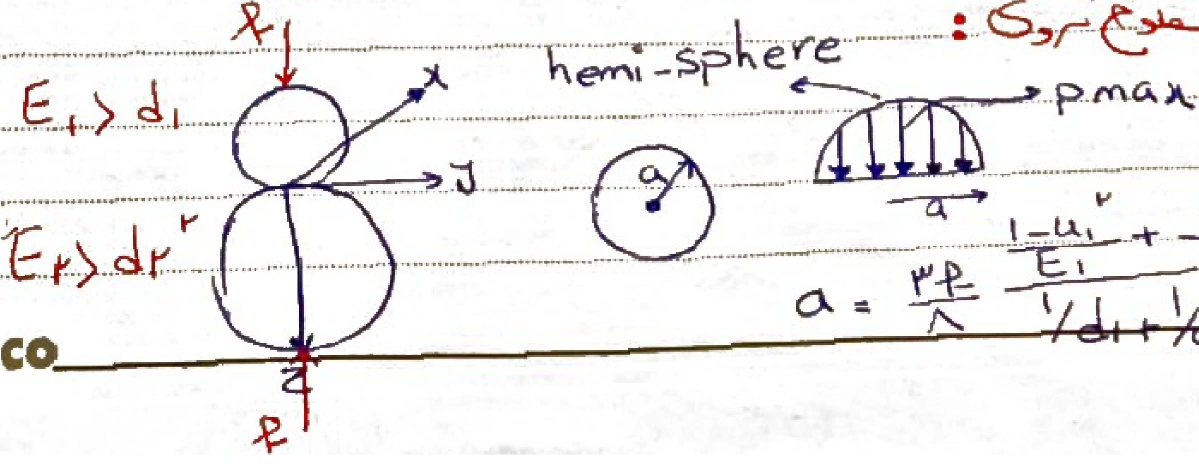
$$P = \rho g h = 13900\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times 120 \times 10^{-3}\,m$$

$$p_i = 19\,kPa \rightarrow \frac{19\times10^3 \times \left(\frac{19}{2}\times10^{-3}\right)^2}{\left(\frac{27}{2}\times10^{-3}\right)^2 - \left(\frac{19}{2}\times10^{-3}\right)^2}\left(1 - \frac{\left(\frac{27}{2}\times10^{-3}\right)^2}{r^2}\right)$$

$$t = \frac{19\times10^3\,\frac{N}{m^2} \times \left(\frac{19}{2}\times10^{-3}\right)^2}{\left(\frac{27}{2}\times10^{-3}\right)^2 - \left(\frac{19}{2}\times10^{-3}\right)^2}\left(1 + \frac{\left(\frac{27}{2}\times10^{-3}\right)^2}{r^2}\right)$$

Hertzian stress:

الف) سطوح کروی:



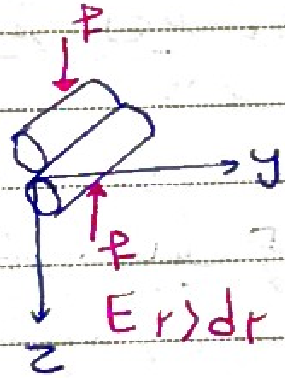
$$a = \frac{3F}{8}\,\frac{\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}}{1/d_1 + 1/d_2}$$

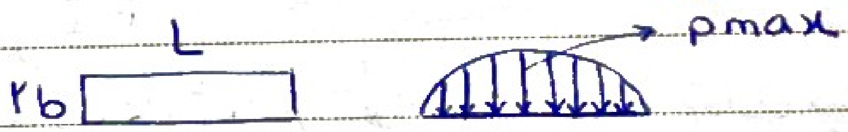
$$p_{max} = \frac{3P}{2\pi a^2}$$

$$\sigma_x = \sigma_y = \sigma_1 = \sigma_2 = -p_{max}\left[\left(1-\frac{z}{a}\right)tg^{-1}\left|\frac{1}{z/a}\right|(1+\nu) - \frac{1}{2\left(1+\frac{z^2}{a^2}\right)}\right]$$

$$\sigma_z = \sigma_3 = \frac{-p_{max}}{1+\frac{z^2}{a^2}} \Rightarrow \tau = \frac{\sigma_{max}+\sigma_{min}}{2}$$

ب) سطح استوانه ای:





$$b = \sqrt{\frac{2P}{\pi L}\,\frac{\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}}{1/d_1+1/d_2}}$$

$$\sigma_x = -2\nu p_{max}\left[\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right]$$

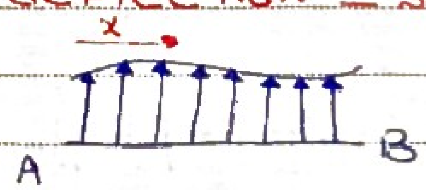
$$\sigma_y = -p_{max}\left[\frac{1+2z^2/b^2}{\sqrt{1+\frac{z^2}{b^2}}} - 2\left|\frac{z}{b}\right|\right]$$

$$\sigma_T = \frac{-p_{max}}{\sqrt{1+\frac{z^2}{b^2}}}$$

SALEH

Subject ______ Year: ______ Month: ______ Date: ______

deflection _ slope :



$q = q(x)$

$$q(x) = \frac{dV}{dx} \qquad V(x) = \frac{dM(x)}{dx}$$

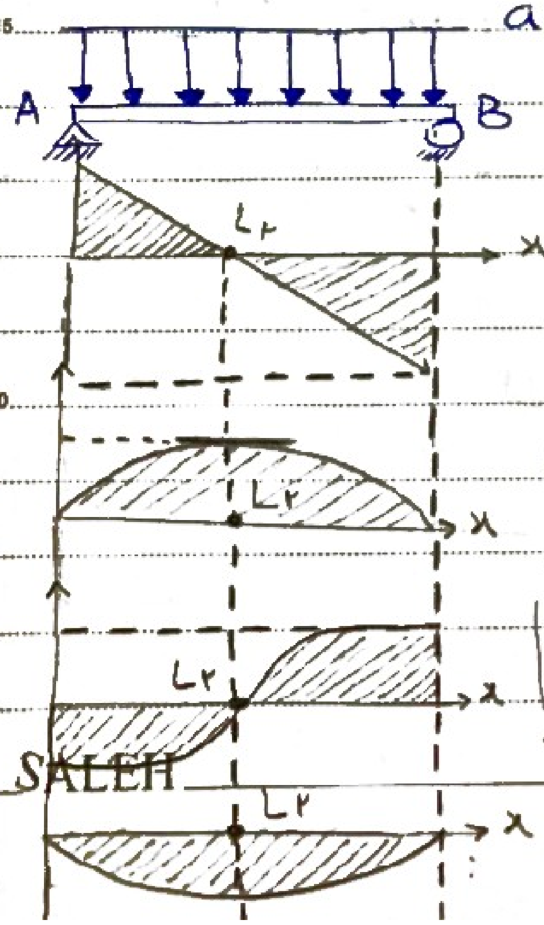
$$\frac{1}{\rho} = \frac{M(x)}{EI} = \frac{y''}{(1+y'^2)^{3/2}} \simeq y''$$

$$y' = \frac{1}{EI}\int M(x)\,dx + C_1$$

$$y = \frac{1}{EI}\iint M(x)\,dx\,dx + Cx + C_2$$

$$\boxed{\frac{d^4y}{dx^4} = \frac{q(x)}{EI}} \longrightarrow \text{معادله دیفرانسیل تیر}$$

مثال) خیز و شیب را در طول بارگذاری شده در شکل با استفاده از معادله دیفرانسیل تیر نقش نمایید؟



$q(x) = \omega_0$

$$-\omega_0 = \frac{dV(x)}{dx} \rightarrow V(x) = -\omega_0 x + C_1$$

$$V(x=0) = \frac{\omega_0 L}{2} \longrightarrow C_1 = \frac{\omega_0 L}{2}$$

$$\boxed{V(x) = -\omega_0 x + \frac{\omega_0 L}{2}}$$

$$M(x) = \frac{-\omega_0}{2}x^2 + \frac{\omega_0 L}{2}x + C_2$$

$$M(x=0) = 0 \longrightarrow \boxed{C_2 = 0}$$

$$\boxed{\frac{M(x)}{EI} = \frac{-\omega_0}{2EI}x^2 + \frac{\omega_0 L}{2EI}x}$$

SALEH

Subject — Year: — Month: — Date:

$$\theta(x) = \frac{-w_0}{6EI}x^3 + \frac{w_0L}{6EI}x^2 + C_3$$

$$y(x) = \frac{-w_0}{24EI}x^4 + \frac{w_0L}{12EI}x^3 + C_3x + C_4$$

$$y(x=0)=0 \longrightarrow C_4 = 0$$

$$y(x=l)=0 \longrightarrow 0 = \frac{-w_0}{24EI}L^4 + \frac{w_0L}{12EI}L^3 + C_3L$$

$$C_3 = \frac{-w_0L}{24EI} \longrightarrow y(x) = \frac{-w_0}{24EI}x^4 + \frac{w_0L}{12EI}x^3 - \frac{w_0L}{24EI}x$$

$$\boxed{y(x) = \frac{-w_0L^4}{24EI}\left(\left(\frac{x}{L}\right)^4 - 2\left(\frac{a}{L}\right)^3 + \left(\frac{x}{L}\right)^2\right)}$$

$$\boxed{y(x) = \frac{-w_0L^4}{24EI}\left\{4\left(\frac{x}{L}\right)^3 - 9\left(\frac{x}{L}\right)^2 + 1\right\}}$$

$$y_{max}(x=1/2) = \frac{-w_0L^4}{24EI}\left\{\frac{1}{16} - \frac{1}{4} + \frac{1}{2}\right\} = \frac{-5w_0L^4}{389EI}$$

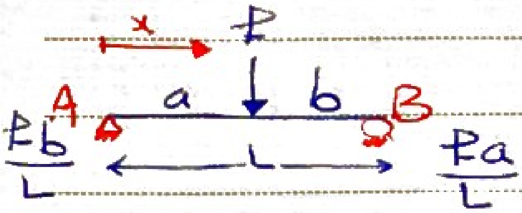
$$\theta_{max}(x=0) = \frac{-w_0L^3}{24EI} \longrightarrow \frac{w_0L^3}{24EI}\ (x=L)$$

$$\theta_{max}(x=1) = \frac{-w_0L^3}{24EI}\{4 - 9 + 1\} = \frac{w_0L^3}{24EI}$$

SALEH

Subject Year: Month: Date:

تعیین خیز و شیب با استفاده از توابع تکین:



$$q(x) = \frac{Pb}{L}\langle x\rangle^{-1} - P\langle x-a\rangle^{-1} + \frac{Pa}{L}\langle x-L\rangle^{-1}$$

$$V(x) = \frac{Pb}{L}\langle x\rangle^{0} - P\langle x-a\rangle^{0} + \frac{Pa}{L}\langle x-L\rangle^{0}$$

$$V(x) = \frac{Pb}{L}\langle x\rangle^{1} - P\langle x-a\rangle^{1} + \frac{Pa}{L}\langle x-L\rangle^{1}$$

$$M(x) = \frac{Pb}{L}x - P\langle x-a\rangle^{-1}$$

$$EI\theta(x) = \frac{Pb}{2L}x^2 - \frac{P}{2}\langle x-a\rangle^{2} + C_1$$

$$EI\,y(x) = \frac{Pb}{6L}x^3 - \frac{P}{6}\langle x-a\rangle^{3} + C_1 x + C_2$$

$y(0)=0 \longrightarrow \boxed{C_2 = 0}$ $\qquad y(L)=0 \longrightarrow 0 = \frac{Pb}{6L} - \frac{P}{6}(L-a) + C_1 L$

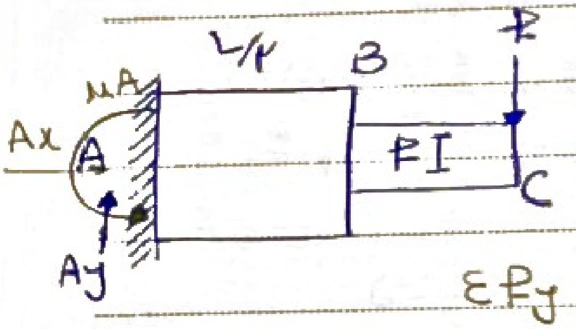
$$C = \frac{-Pb}{6L}L^2 - \frac{Pb^3}{6L}$$

$$y(x) = \frac{1}{EI}\left\{\frac{Pb}{6L}(x^3 - L^2x) - \frac{P}{6}\langle x-a\rangle^3 - \frac{Pb^3}{6L}x\right\}$$

Subject ۹۸,۹,۱۳

Year: Month: Date:

مثال) تغییرات خیز و شیب را در سیستم بارگذاری شده در شکل زیر ترسیم نمایید.



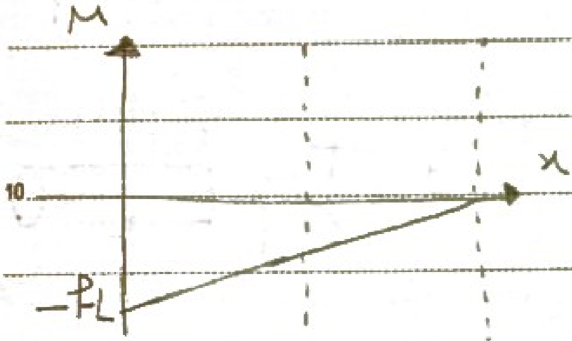
$\Sigma F_x = 0 \longrightarrow A_x = 0$

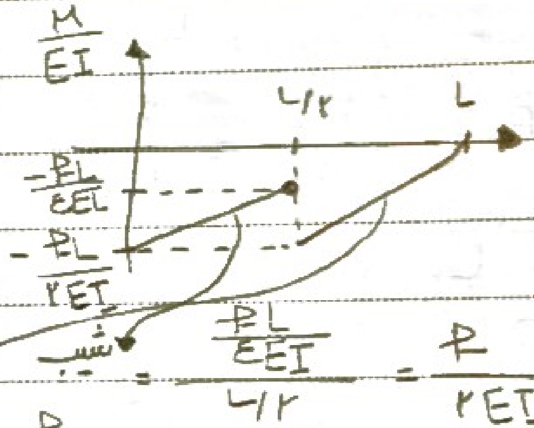
$\Sigma F_y = 0 \longrightarrow A_y = P$

$\Sigma M_A = 0 \longrightarrow M_A - PL = 0$

$\Sigma F_y = 0 \longrightarrow V - P = 0 \longrightarrow V = P$

$\Sigma M_P = 0 \longrightarrow M + PL - Px = 0 \Rightarrow M = PL - Px$





$\text{شیب} = \dfrac{\frac{PL}{4EI}}{L/2} = \dfrac{P}{2EI}$

$\text{شیب} = \dfrac{P}{EI}$

$$\frac{M(x)}{EI} = \frac{P}{2EI}x - \frac{PL}{2EI} - \frac{PL}{4EI}\langle x - L/2\rangle^0 + \frac{P}{2EI}\langle x - L/2\rangle^1$$

$$\frac{dy}{dx} = \frac{P}{4EI}x^2 - \frac{PL}{2EI}x - \frac{PL}{4EI}\langle x - L/2\rangle^1 + \frac{P}{4EI}\langle x - L/2\rangle^2 + C_1$$

$$\frac{dy}{dx}(x=0) = 0 \longrightarrow C_1 = 0$$

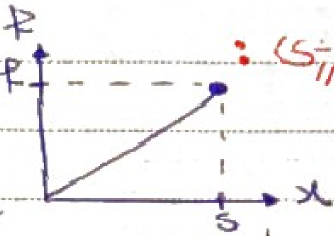
$$y = \frac{P}{12EI}x^3 - \frac{PL}{4EI}x^2 - \frac{PL}{8EI}\langle x - L/2\rangle^2 + \frac{P}{12EI}\langle x - L/2\rangle^3 + C_2$$

$$y(x=0) = 0 \longrightarrow C_2 = 0$$

SALEH

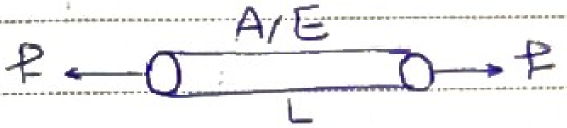
روش انرژی :

$P = K\delta$



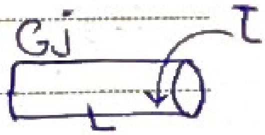
$U = \int P\,dx = \int Kx\,dx = \frac{1}{2}Kx^2$

$U = \frac{1}{2}K\delta^2 = \frac{1}{2}K\frac{P^2}{K^2} = \frac{P^2}{2K} \Rightarrow U = \frac{1}{2}P.\delta$

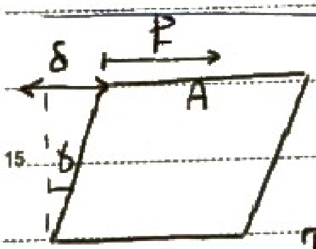


در $\delta = \frac{PL}{AE} \longrightarrow P = \frac{AE}{L}\delta$

$U = \frac{1}{2}P\,\frac{PL}{AE} = \frac{P^2L}{2AE}$ $\longrightarrow$ $\theta = \frac{TL}{GJ}$



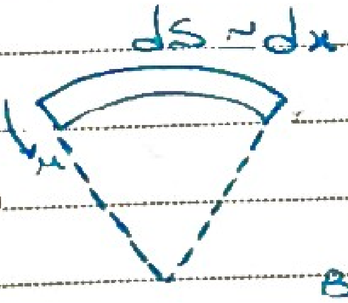
$$U = \frac{T^2L}{GJ}$$



$U = \frac{P^2L}{2AG}$

$\tau = G\gamma,\ \gamma = \frac{\delta}{L}$

$ds \simeq dx$



$dU = \frac{1}{2}M\,d\theta = \frac{1}{2}M\,ds/\rho$

$1/\rho = M/EI \longrightarrow dU = \frac{1}{2}\frac{M^2dx}{EI}$

$U_{Bending} = \int \frac{M^2dx}{2EI}$

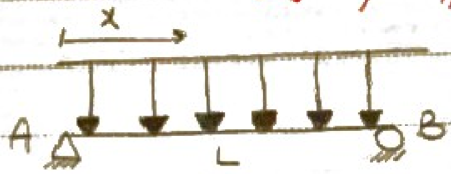
Shear Bending

$U = \int \frac{\alpha V^2}{2AG}dx$

$\tau = \frac{VQ}{Ib}$

Subject ______ Year: ______ Month: ______ Date: ______

حل مثال اول جلسه قبل با استفاده از انرژی کرنشی :



$$V(x) = -w_0 x + \frac{w_0 L}{2}$$

$$M(x) = \frac{-w_0}{2} x^2 + \frac{w_0 L}{2} x$$

$$U_{Bending} = \int \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^L \left[\frac{w_0 L^2}{2} \left(\left(\frac{x}{L}\right)^2 + \left(\frac{x}{L}\right) \right) \right]^2 dx$$

$$= \frac{w_0^2 L^4}{8EI} \int_0^L (x/L)^4 + 2(x/L)^3 + (x/L)^2 dx$$

$$= \frac{w_0^2 L^4}{8EI} \left\{ \frac{L}{5} + 2\frac{L}{4} + \frac{1}{3} \right\} = \frac{31 w_0^2 L^5}{240 EI}$$

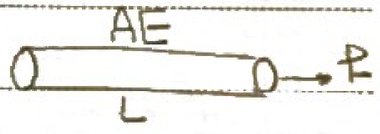
$$U_{shearBending} = \int \frac{C V^2}{2AG} dx = \int \frac{C}{2AG} \times \left[-wd \left(\frac{-x}{L} \right) - \frac{1}{2} \right) \Big]^2 dx$$

$$= \frac{C w_0^2 L^2}{2AG} \int_0^L \left(\frac{x}{L}\right)^2 - \left(\frac{x}{L}\right) + 1/4 \, dx$$

$$= \frac{C w_0^2 L^2}{2AG} \left\{ \frac{1}{3} - \frac{L}{2} + \frac{L}{4} \right\}$$

$$\rightarrow U_{shear\ Bending} = \frac{C w_0 L^3}{24 AG}$$

$$U = \frac{P^2 L}{2AE}, \quad \delta = \frac{PL}{AE} \longrightarrow \delta = \frac{\partial U}{\partial P}$$



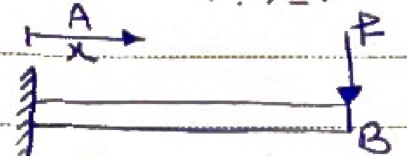
Castigliano's thearem

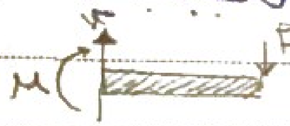
SALEH

Subject

Year: Month: Date:

مثال) با استفاده از قضیه قبل جابجایی نقطه B در تیر بارگذاری شده در شکل را بدست آورید؟



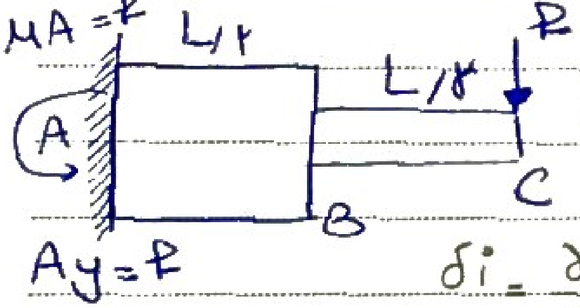


$$\Sigma M_P = 0 \longrightarrow M + Px = 0 \longrightarrow M = -Px$$

$$U_{Bending} = \int \frac{M^2 dx}{2EI} = \frac{1}{2EI}\int_0^L (-Px)^2 dx = \frac{P^2}{2EI} \times \frac{L^3}{3}$$

$$= \frac{P^2L^3}{6EI} \longrightarrow \delta_B = \frac{\partial U}{\partial P} = \frac{PL^3}{3EI}$$

مثال) با استفاده از قضیه قبل در تیر نشان داده شده در شکل جابجایی نقاط C و B را نشان دهید؟



$$M = -Px$$

$$U_{Bending} = \int \frac{M^2 dx}{2EI}$$

$$\delta_i = \frac{\partial U}{\partial P_i} = \int \frac{1}{EI} M \left(\frac{\partial M}{\partial P_i}\right) dx$$

$$U = \int \frac{P^2 dx}{2AE} \longrightarrow \delta_i = \frac{\partial U}{\partial P_i} = \int \frac{1}{AE} P \left(\frac{\partial P}{\partial P_i}\right) dx$$

$$U_{torision} = \int \frac{T^2 L}{2GJ} \longrightarrow \theta_i = \frac{\partial U}{\partial T_i} = \int \frac{1}{GJ} T \left(\frac{\partial T}{\partial T_2}\right) dx$$

$$\delta_C = \int_0^L \frac{1}{EI}(-Px)(-x)\,dx = \int_0^{L/2} \frac{1}{2EI}(-Px)(-x)\,dx + \int_{L/2}^{L} \frac{1}{2EI}(-Px)(-x)\,dx = \frac{P}{EI} \times \frac{1}{3} \times \left(\frac{L}{2}\right)^3 + \frac{P}{2EI} \times \frac{1}{3}$$

$$\times \left(L^3 - \left(\frac{L}{2}\right)^3\right) = \frac{PL^3}{24EI} + \frac{7PL^3}{48EI} = \frac{9PL^3}{48EI}$$

$$M = -Px - Q_1\left(x - \frac{L}{2}\right) \qquad \frac{L}{2} < x < L$$

SALEH $\delta_B = \int \frac{1}{EI} M \left(\frac{\partial M}{\partial Q_i}\right) dx = \int_{L/2}^{L} \frac{1}{EI}(-Px)\,dx$

$$\delta_B = \frac{P}{2EI}\left\{\frac{7L^3}{24} - \frac{3L^3}{16}\right\} = \frac{5PL^3}{96EI}$$

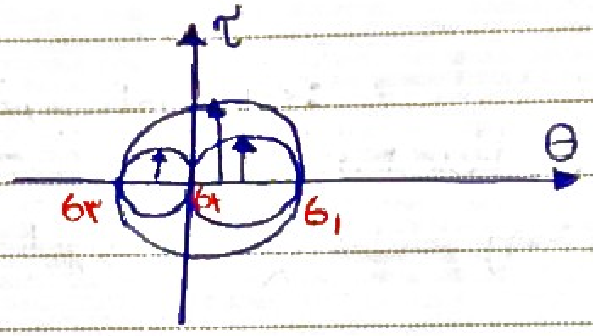
Subject ۹۸/۹/۲۰ Year: Month: Date:

□ معیارهای شکست:

الف) استاتیکی:

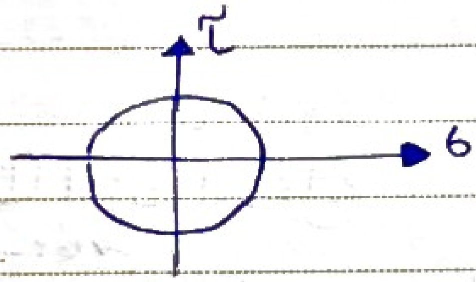
① تنش برشی بیشینه — Maximum shean stress

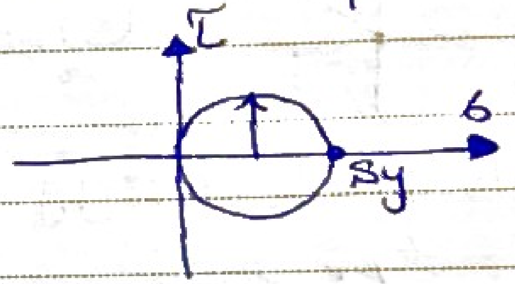
تنش‌های اصلی $\sigma_1 > \sigma_2 > \sigma_3$



$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$





S_y = yield stress   تنش تسلیم

$$\frac{S_y}{2} = S_{sy}$$

$\tau_{max} \geq S_{sy}$ شرط شکست

$$\frac{\sigma_1 - \sigma_3}{2} \geq \frac{S_y}{2} \longrightarrow \boxed{\sigma_1 - \sigma_3 \geq S_y}$$

در حالت دوبعدی (σ_A , σ_B) تنش‌های اصلی است.

الف) $\sigma_A > \sigma_B > 0$

$\sigma_A - 0 \geq S_y \longrightarrow S_y \longrightarrow \sigma_A \geq S_y$ $\boxed{\sigma_A \geq S_y/n}$

ب) $\sigma_A > 0 > \sigma_B$

$\boxed{\sigma_A - \sigma_B \geq S_y/n}$

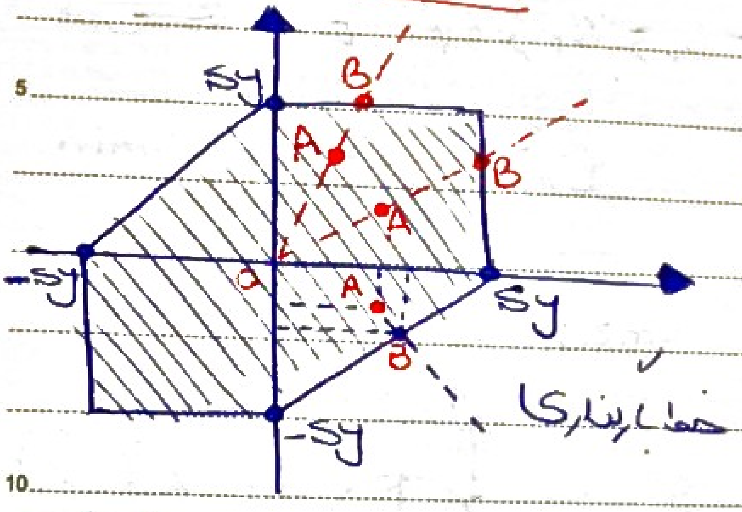
SALEH

ج)

$\sigma_A > \sigma_B$

$0 - \sigma_B \geq S_y$

$\boxed{\sigma_B \leq -S_y / n}$

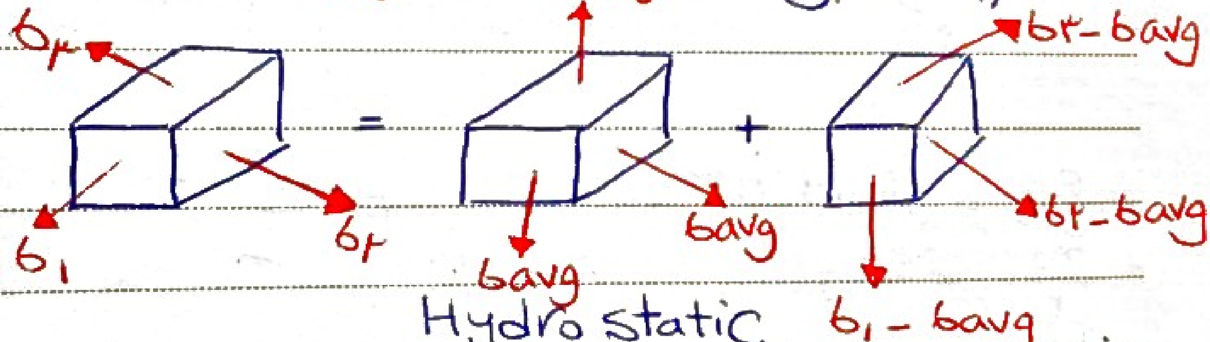


ضریب اطمینان:

n = Factor of safety

$S_y \longrightarrow \frac{S_y}{n}$, $n \geq 1$

Distorsion energy ② انرژی اعوجاج



$\sigma_{avg} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$

U انرژی کرنشی $= \frac{1}{2} FS$ انرژی کرنشی بر حجم $u = \frac{U}{V}$

$u = \frac{1}{2} \frac{FS}{AL} = \frac{1}{2} \sigma \varepsilon$

(1) $u = \frac{1}{2} (\sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3)$

$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\nu \sigma_2}{E} - \frac{\nu \sigma_3}{E}$

$\varepsilon_2 = \sigma_3/E - \nu\sigma_1/E + \sigma_2 E_2$

$\varepsilon_3 = \sigma_3/E - \nu\sigma_1/E - \nu\sigma_2/E$

$u_1 = 1/2E\,(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_1))$

(۲) $u_2 = \frac{3\sigma_{avg}^2}{2E}(1 - 2\nu)$

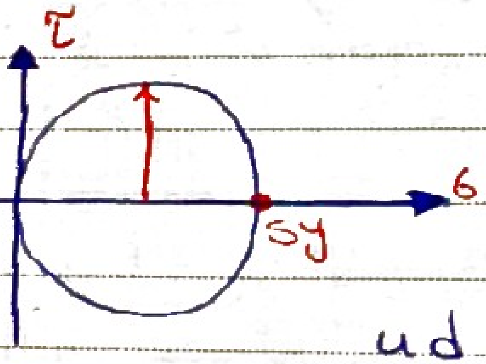
$u_2 = \frac{1-2\nu}{6E}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\sigma_1\sigma_2 + 2\sigma_2\sigma_3 + 2\sigma_1\sigma_3)$

$u_d = u_1 - u_2 = (\sigma_1^2 + \sigma_2^2 + \sigma_3^2)\left(\frac{1}{6E} - \frac{1-2\nu}{6E}\right) +$
$(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\left(\frac{-\nu}{E} - \frac{1-2\nu}{3E}\right)$

$u_d \longrightarrow \frac{1+\nu}{3E}\left[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$

$= \frac{1+\nu}{3E}\left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 (\sigma_3-\sigma_1)^2}{2}\right]$

تست کشش:



$\sigma_1 = S_y$

$\sigma_2 = \sigma_3 = 0$

$u_d = \frac{1+\nu}{3E} S_y^2$

$S_y \geq \sqrt{\frac{(\sigma_1-\sigma_2) + (\sigma_2-\sigma_3)(\sigma_1-\sigma_3)}{2}}$ → von-mises stress (σ')

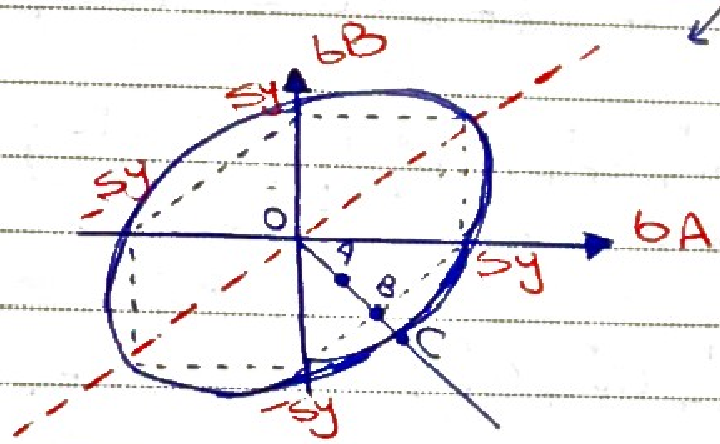
$n = \frac{S_y}{\sigma'} \rightarrow \sigma' = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{zy}^2 + \tau_{xz}^2)}$

□ در حالت دوبعدی σ_A و σ_B (تنش‌های اصلی)

$$\sigma' = \sqrt{\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2}$$

$$\sigma' = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2}$$

$$\longrightarrow \boxed{\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2 = S_y^2}$$ معادله بیضی



مثال) اگر تنش‌های اصلی در یک قطعه مطابق حالت‌های زیر باشند و تنش تسلیم $S_y = 100\ mpa$ باشد، ضریب اطمینان را با استفاده از تنش‌های برشی max و تنش‌های اعوجاجی به دست آورید؟

الف) ۰ و ۷۰ و ۷۰ mpa
ب) ۳۰ و ۷۰ و ۰ mpa
ج) ۰ و +۷۰ و +۳۰ mpa
د) -۳۰ و -۷۰ و ۰ mpa
ه) ۳۰ و ۳۰ و ۳۰ mpa

(الف) $\tau_{max} = \frac{۷۰ - ۰}{۲} = ۳۵\ mpa$

$SSy = Sy/۲ = ۱۰۰/۲ = ۵۰$

$n = SSy/\tau = \frac{۵۰}{۳۵} = ۱٫۴۲۸$

$\sigma' = \sqrt{۷۰^۲ - ۷۰^۲ + ۷۰^۲} = ۷۰$

$n = Sy/\sigma' = \frac{۱۰۰}{۷۰} = ۱٫۴۲۸۵۷$

(ب) $\tau_{max} = \frac{۷۰ - ۰}{۲} = ۳۵$

$SSy = Sy/۲ = ۵۰ \rightarrow n = SSy/\tau = ۱٫۴۲۸$

$\sigma' = \sqrt{۷۰^۲ - (۷۰ \times ۳۰) + ۳۰^۲} = ۱۰\sqrt{۳۷}$

SALEH $n = Sy/\sigma' = \frac{۱۰۰}{۱۰\sqrt{۳۷}} = ۱٫۶٤$

(ج) $\tau_{max} = \frac{۷۰+۳۰}{۲} = ۵۰$

$S_{sy} = S_y/۲ = ۵۰$

$n = \frac{S_{sy}}{\tau_{max}} = \frac{۵۰}{۵۰} = ۱$　　$\sigma' = \sqrt{۷۰^۲ - ۷۰\times(-۳۰) + (-۳۰)^۲} = ۱۰\sqrt{۷۹}$

$n = S_y/\sigma' = \frac{۱۰۰}{۱۰\sqrt{۷۹}} = ۱.۱۲$

(د) $\tau_{max} = ۰+۷۰/۲ = ۳۵$　　$S_{sy} = ۵۰$

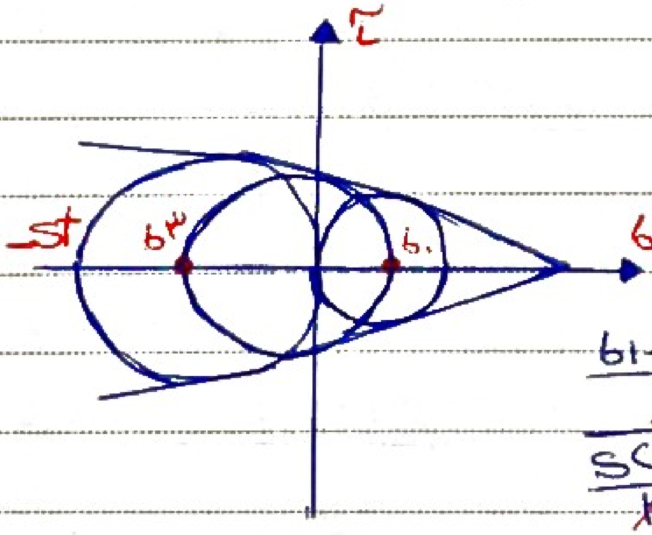
$n = \frac{۵۰}{۳۵} = ۱.۴۲$　　$\sigma' = \sqrt{(-۷۰)^۲ - (-۳۰\times -۷۰) + (-۳۰)^۲} = ۱۰\sqrt{۳۷}$

$n = \frac{۱۰۰}{۱۰\sqrt{۳۷}} = ۱.۶۴$

(هـ) $\tau_{max} = ۰$　　$n = ۵۰/۰$　　عدد/۰ → ∞

$\sigma' = \sqrt{۳۰^۲ - ۳۰^۲ + ۳۰^۲} = ۰$　　$n = \frac{۱۰۰}{۰}$ → ∞

Coulomb-mohr (۳)



$$\frac{\frac{\sigma_1-\sigma_3}{2} - \frac{S_t}{2}}{\frac{S_c}{2} - \frac{\sigma_1-\sigma_3}{2}} = \frac{\frac{S_t}{2} - \frac{\sigma_1+\sigma_3}{2}}{\frac{S_t}{2} + \frac{S_c}{2}}$$

$S_t/2$

$\Rightarrow [(\sigma_1-\sigma_3) - S_t][S_t + S_c] = [S_t - (\sigma_1+\sigma_3)][C_S - S_t]$

$\sigma_1 S_t - \sigma_3 S_t - \sigma_3 S_c - S_t^2 - S_t S_c =$

$S_t S_c - S_t^2 - \sigma_1 S_c + \sigma_1 S_t - \sigma_3 S_c + \sigma_3 S_t$

$\Rightarrow 2\sigma_1 S_c - 2\sigma_3 S_t = 2 S_t S_c$

SALEH

$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_t} = 1$

$\sigma_A > \sigma_B > 0 \rightarrow \sigma_A / S_t = 1 \rightarrow \sigma_A = S_t$ (حالت الف)

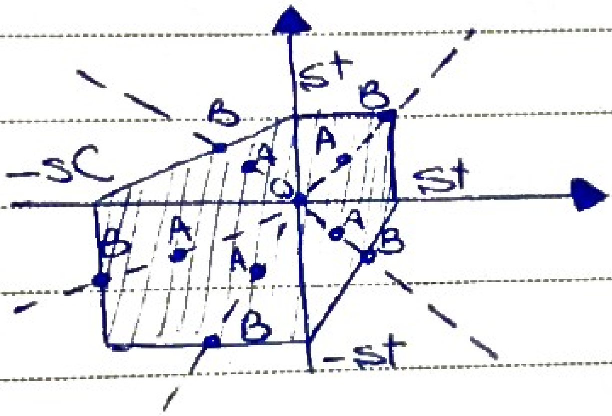
$\boxed{\sigma_A = S_t / n}$

ب) $\sigma_A > 0 > \sigma_B$

$\sigma_A / S_t - \sigma_B / S_C = 1$ $\boxed{\sigma_A / S_t - \sigma_B / S_C = 1/n}$

ج) $0 > \sigma_A > \sigma_B$

$0 - \frac{\sigma_B}{S_C} = 1 \rightarrow \sigma_B = S_C \rightarrow \boxed{\sigma_B = \frac{S_C}{n}}$



Subject ۹۸,۹,۲۷ Year: Month: Date:

$\sigma_y = -40\,MPa$, $\tau = 30\,MPa$, $\sigma_x = 50\,MPa$

$S_y = 150\,MPa$ $(n)_{MSS} = ?$ $(n)_{DE} = ?$

$\sigma_A = \sigma_{avg} + R$

$\sigma_B = \sigma_{avg} - R$

$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{(+50)+(-40)}{2} = 10$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \rightarrow R = \sqrt{\left(\frac{50-(-40)}{2}\right)^2 + 30^2} = 50\,MPa$

$\sigma_A = 10 + 50 = 60$ $\sigma_B = 10 - 50 = -40$

$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{60 - 40}{2} = 50\,MPa$

$(n)_{MSS} = \frac{S_{sy}}{\tau_{max}} = \frac{75}{50} = 1.5$

$(n)_{DE} = \frac{S_y}{\sigma'} \rightarrow \sigma' = \sqrt{\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2}$

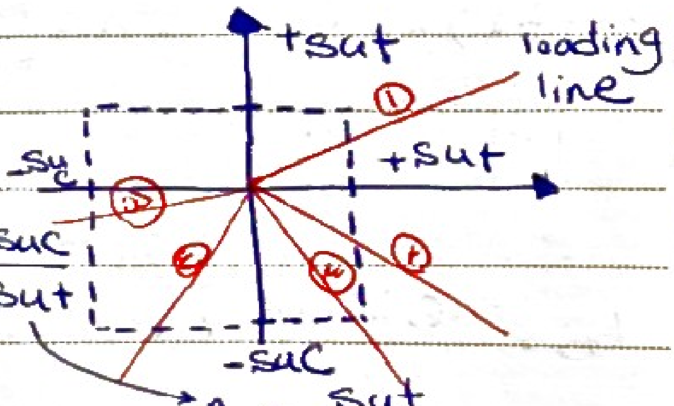
$= \sqrt{(60)^2 - (60 \times -40) + (-40)^2} = \sqrt{7600} = 10\sqrt{76}$

$(n)_{DE} = \frac{150}{10\sqrt{76}} = \ldots$

Static Failure (Brittle Materials) سوال تدریس نشده

1 Maximum normal stress (MNS)

su: ultimate stress



A) $\sigma_A > \sigma_B > 0 \rightarrow n = \frac{S_{ut}}{\sigma_A}$

B) $\sigma_A > 0 > \sigma_B \rightarrow |\sigma_B/\sigma_A| < \frac{S_{uc}}{S_{ut}}$ → $n = \frac{S_{ut}}{\sigma_A}$

C) $\sigma_A > 0 > \sigma_B$

$\rightarrow \left|\frac{\sigma_B}{\sigma_A}\right| > \frac{S_{uc}}{S_{tu}}$

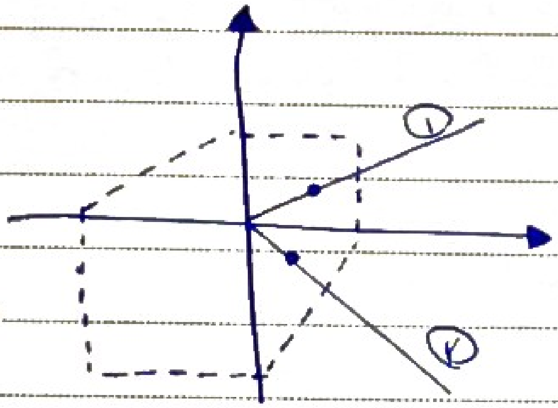
$n = \frac{-S_{uc}}{\sigma_B}$

SALEH

ع) $0 > \sigma_A > \sigma_B \longrightarrow \left|\frac{\sigma_B}{\sigma_A}\right| > 1 \longrightarrow n = \frac{-S_{uc}}{\sigma_B}$

ه) $0 > \sigma_A > \sigma_B \longrightarrow \left|\frac{\sigma_B}{\sigma_A}\right| < 1 \longrightarrow n = \frac{-S_{uc}}{\sigma_B}$

۲ Coulomb _ Mohr Brittle

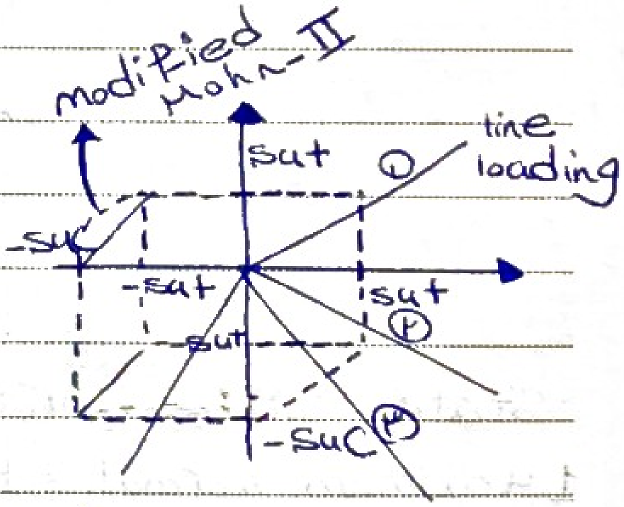


الف) $\sigma_A > \sigma_B > 0 \longrightarrow n = \frac{S_{ut}}{\sigma_A}$

ب) $\sigma_A > 0 > \sigma_B \longrightarrow \frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n}$

ج) $0 > \sigma_A > \sigma_B \longrightarrow n = \frac{-S_{uc}}{\sigma_B}$

۳ Modified Mohr _ I



الف) $\sigma_A > \sigma_B > 0 \longrightarrow n = \frac{S_{ut}}{\sigma_A}$

ب) $\sigma_A > 0 > \sigma_B \longrightarrow n = \frac{S_{ut}}{\sigma_A}$

$\left|\frac{\sigma_B}{\sigma_A}\right| < 1$

ج) $\sigma_A > 0 > \sigma_B \longrightarrow n = \frac{S_{uc} - S_{ut}}{S_{uc} + S_{ut}}\sigma_A - \frac{\sigma_B}{S_{uc}} = \frac{1}{n}$

$\left|\frac{\sigma_B}{\sigma_A}\right| > 1$

$\sigma_B = \frac{S_{uc} - S_{ut}}{S_{ut}}\sigma_A - S_{uc} \longrightarrow \frac{S_{uc} - S_{ut}}{S_{uc} + S_{ut}}\sigma_A - \frac{\sigma_B}{S_{uc}} = 1$

د) $0 > \sigma_A > \sigma_B \longrightarrow n = \frac{-S_{uc}}{\sigma_B}$

Modified Mohr – II → ③ $\sigma_A > 0 > \sigma_B \rightarrow \left|\frac{\sigma_B}{\sigma_A}\right| > 1$

$$\rightarrow \frac{n\sigma_A}{S_{ut}} + \frac{(n\sigma_B + S_{ut})^2}{(S_{uc} - S_{ut})^2} = \frac{1}{}$$

$$(\sigma_B + S_{ut})^2 = C(\sigma_A - S_{ut})$$

$$(-S_{uc} + S_{ut})^2 = C(0 - S_{ut})$$

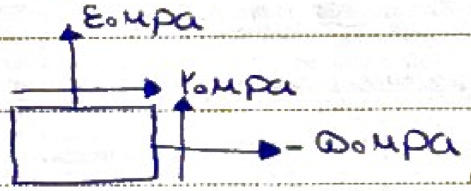
$$\Rightarrow C = \frac{-(S_{uc} - S_{ut})^2}{S_{ut}}$$

$$\Rightarrow (\sigma_B + S_{ut})^2 = \frac{-(S_{uc} - S_{ut})^2}{S_{ut}}(\sigma_A - S_{ut})$$

$$\Rightarrow \frac{\sigma_A - S_{ut}}{S_{ut}} - \frac{(\sigma_B + S_{ut})^2}{(S_{uc} - S_{ut})^2} = 0$$

$$\frac{\sigma_A}{S_{ut}} + \frac{(\sigma_B + S_{ut})^2}{(S_{uc} - S_{ut})^2} = 1$$

مثال) در المان تنش نشان داده شده از یک ماده ترد در حالت استاتیکی ضریب اطمینان را از معیارهای MNS / Coulomb Mohr / Modified Mohr I / Modified Mohr II بدست آورید.



$S_{ut} = 150$

$S_{uc} = 200$

$$\sigma_A = \sigma_{avg} + R = -5 + 20.61 = 15.6$$

$$\sigma_B = \sigma_{avg} - R = -5 - 20.61 = -25.6$$

$$\sigma_{avg} = \frac{\sigma_A + \sigma_B}{2} = \frac{-50 + 40}{2} = -5$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-50-(-40)}{2}\right)^2 + 20^2} = \sqrt{425} = 20.61$$

$$\rightarrow \frac{15.6}{150} - \frac{25.6}{200} = \frac{1}{n} \rightarrow n = 4.31$$

$$\frac{200 - 150}{200 \times 150} 15.6 + \frac{25.6}{200} = \frac{1}{n} \rightarrow n = 6.49$$

$$\frac{n\,15.6}{150} + \frac{(25.6n + 150)^2}{(200 - 150)} = 1 \rightarrow n = \dots$$

SALEH

Subject

Year: Month: Date:

$$\frac{SUL}{SUT} = \frac{۲۰۰}{۱۵۰} = \frac{۴}{۳} = ۱,۳۳ \rightarrow \frac{\Delta A}{\Delta B} > ۱,۳۳$$

۱,۳۶

$$n = \frac{-SUL}{\Delta B} = \frac{-۲۰۰}{-۲۵,۴} = ۷,۸۱$$